phys 221.
Lecture 5.

- Gauss' Law.

$$\nabla \cdot \vec{D} = \rho_v \qquad (4.26)$$

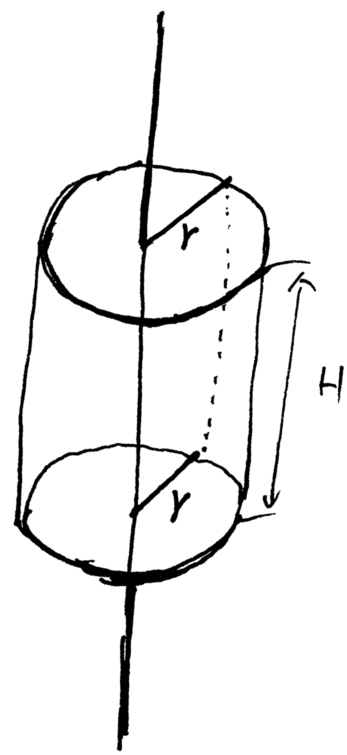
$$\oint_S \vec{D} \cdot d\vec{s} = Q \qquad (4.29)$$

$\vec{D} = \varepsilon \vec{E} = \varepsilon_0 \varepsilon_r \vec{E}$ for istropic and uniform medium

ρ_v — free charge per unit volume.

e.g: A Long straight wire with a uniform linear charge density ρ_ℓ.

Find $\vec{E}$ field.



Gaussian surface.

by symmetry: $\vec{E} = \hat{r} E_r$.

$\vec{D} \cdot d\vec{s} = 0$ on top and bottom.

E_r = const on lateral surface $= E$

$$Q = \oint_S \vec{D} \cdot d\vec{s} = \oint_S \varepsilon_0 \vec{E} \cdot d\vec{s}$$

use cylindrical coord.

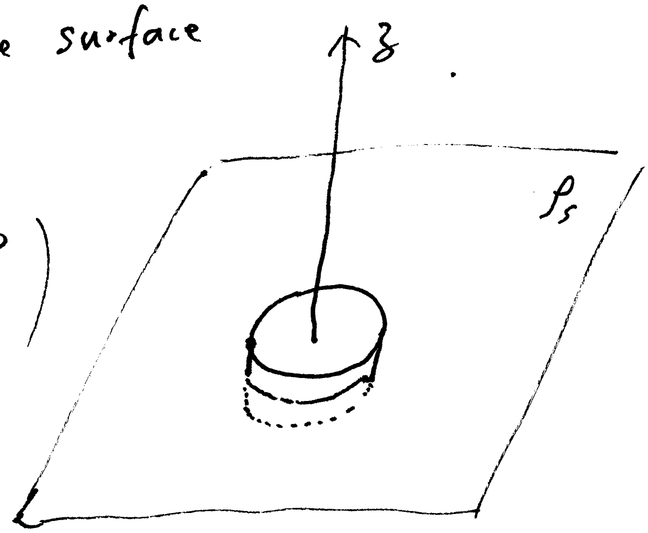
$$= \varepsilon_0 E_r \cdot 2\pi r \cdot H.$$

$$E_r = \frac{Q}{2\pi \varepsilon_0 H} = \frac{\rho_\ell}{2\pi \varepsilon_0}.$$

(same as previous example)

e.g. For a large flat sheet of charge with a uniform area density of charge ρ_s.

Find $\vec{E}$-field near the surface.

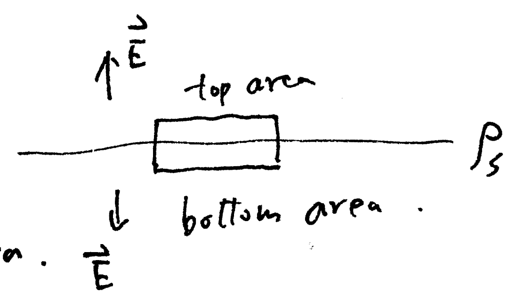


by symmetry.

$\vec{E}_x = 0$. $E_y = 0$. $\begin{pmatrix} E_r = 0 \\ E_\phi = 0 \end{pmatrix}$

Thin drum-shaped Gaussian surface.

$$\oint_S \vec{D} \cdot \vec{ds} = \varepsilon_0 \oint_S \vec{E} \cdot \vec{ds} = Q.$$



Since $E_r = 0$

$\vec{E} \cdot \vec{ds} = 0$ on lateral area.

on top and bottom : $\vec{E} \parallel \vec{ds}$. also, $E = \text{const}$.

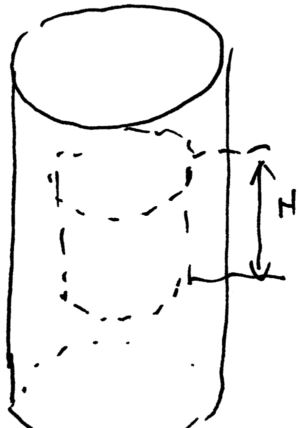
$$\therefore Q = \varepsilon_0 \oint_S \vec{E} \cdot \vec{ds} = \varepsilon_0 \left(E A_{Top} + E A_{bottom} \right)$$

$$= 2 \varepsilon_0 E A.$$

$$E = \frac{Q}{2 \varepsilon_0 A} = \frac{\rho_s}{2 \varepsilon_0}$$

(same as previous example)

3(5/15). A very long solid cylinder with a radius R is uniformly charged. The charge density (charge per unit volume) is ρ. Figure A below depicts its cross section.
(a) Determine the electric field E at point P. The distance from the center of the rod to point P is 2R/3.
(b) If the rod has a cylindrical hole along the rod, with a radius R/2, as shown in figure B below, determine the electric field at point P(2R/3, 0). Note that the centre of the rod, the centre of the hole and point P are all on the x-axis.
(c) If the cylindrical hole inside the rod has a radius w and the centre of the hole is located at (a, b), find the electric field at an arbitrary point (x, y) inside the hole.



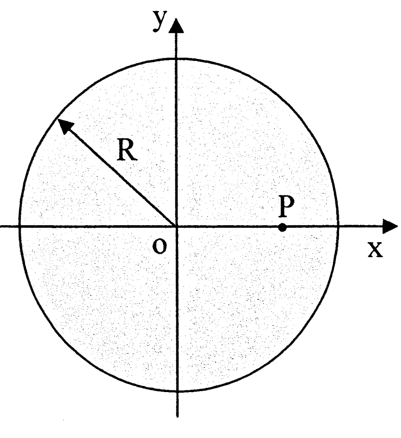


Figure A

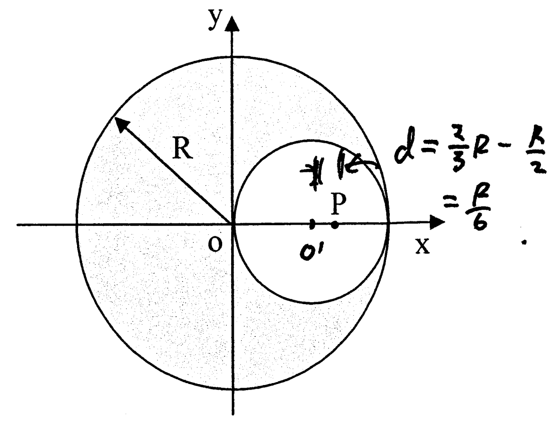


Figure B

(a) use Gauss' law.

$$\frac{Q}{\varepsilon_0} = \oint_S \vec{E}\cdot d\vec{s} = \int_{\text{lateral}} \vec{E}\cdot d\vec{s} = E_r \cdot 2\pi r H = \frac{\rho \pi r^2 H}{\varepsilon_0}.$$

$$\therefore \vec{E} = \hat{r} E_r = \hat{r}\frac{\rho r}{2\varepsilon_0} = \frac{\rho \vec{r}}{2\varepsilon_0} \quad \left(E_z = 0,\ E_\phi = 0 \text{ from symmetry} \right)$$

$$= \hat{r}\frac{\rho R}{3\varepsilon_0} \quad \left(r = \frac{2R}{3} \right)$$

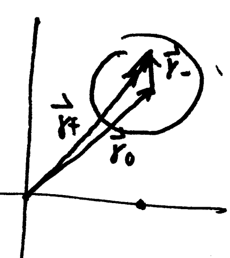
(b). $\vec{E} = \vec{E}_+ + \vec{E}_-$.

at P, $\hat{r} = \hat{x}$.

$$\vec{E} = \hat{x}\left(\frac{\rho R}{3\varepsilon_0} - \frac{\rho}{2\varepsilon_0}\cdot\frac{R}{6} \right) = \hat{x}\frac{\rho R}{\varepsilon_0}\left(\frac{1}{3} - \frac{1}{12} \right) = \hat{x}\frac{\rho R}{4\varepsilon_0}$$

(c). $\vec{E} = \vec{E}_+ + \vec{E}_- = \left(\frac{\rho \vec{r}_+}{2\varepsilon_0} - \frac{\rho \vec{r}_-}{2\varepsilon_0} \right)$

$$= \frac{\rho}{2\varepsilon_0}(\vec{r}_+ - \vec{r}_-) = \frac{\rho}{2\varepsilon_0}\vec{r}_0 = \frac{\rho}{2\varepsilon_0}(\hat{x}a + \hat{y}b)$$



$\vec{r}_0$ — center of hole.