Physics 221

Lecture 8

Ref: 4-5, 4-6, 4-7, 4-8, 4-9, 4-10, 4-11.

- Electric scalar potential

  from coulomb's law: $\boxed{V(\vec{R}) = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{R'} + C}$

  (4.48).

- $V(\vec{R})$ can be ~~solved~~ determined by solving its differential equation.

  In electrostatics, $\vec{E} = -\nabla V$.

  since $\nabla \cdot \vec{E} = \frac{\rho_v}{\varepsilon}$ (in medium, ε) (ρ_v — free charge density.

  $\nabla \cdot (\nabla V) = -\frac{\rho_v}{\varepsilon}$.

  i.e. $\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon}}$. Poisson's equation. (4.60)

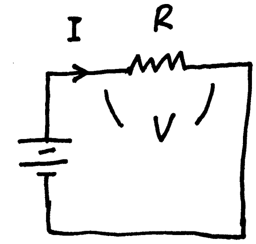
  when there is no free charge, $\rho_v = 0$.

  $\boxed{\nabla^2 V = 0}$ (Laplace's equation). (4.61)

Note: In general, (4.60) and (4.61) can be solved along with boundary conditions.

- Ohm's law.

In a circuit: $R = \frac{V}{I}$.



R — resistance

$$I = \frac{1}{R} \cdot V$$

OR: $I = GV$ G — conductance. (in S). (Siemens)

In a field: $\boxed{\vec{J} = \sigma \vec{E}}$ (4.67) point form of ohm's law.

$\vec{J}$ — current density

σ — conductivity

$\vec{E}$ — electric field.

(very useful law in addition to Maxwell's eqs.)

(for conductors: $\sigma \neq 0$ can be poor conductors!)

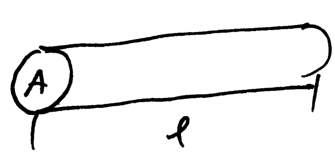
- σ — material dependent. (in S/m):

silver 6.2×10^{7}
copper 5.8×10^{7}
pure Silicon 4.4×10^{-4}
glass 10^{-12}.

$\sigma = \frac{1}{\rho}$.

ρ — resistivity (Ω/m)

$$R = \rho \frac{l}{A} = \frac{l}{\sigma A}.$$



- Perfect conductor: $\sigma = \infty$, $\rho = 0$

$$\therefore \vec{E} = 0.$$

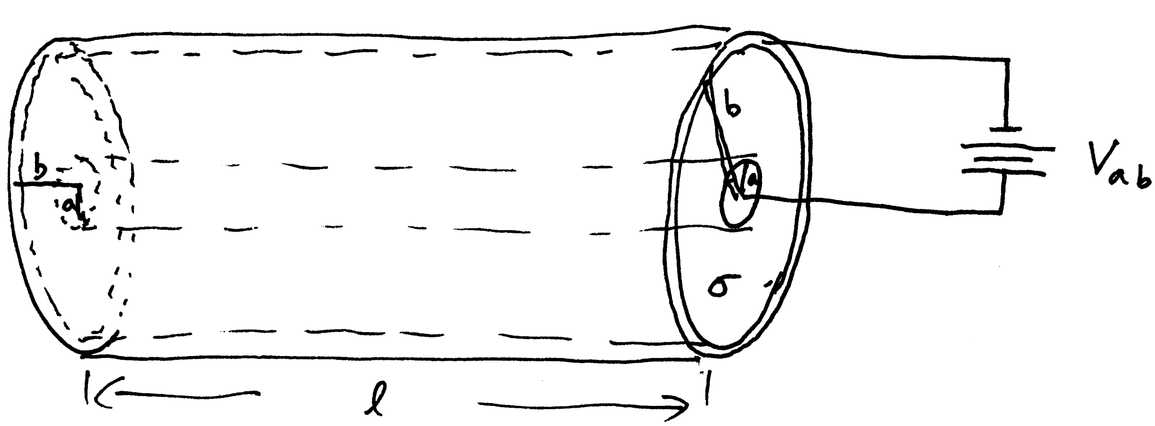
- Perfect dielectric: $\sigma = 0$, $\vec{J} = 0$. (perfect insulator).

- Two kinds of charge carriers: + and −.
  (hole) (electron).

average drift velocity: $\sim 10\ \mu m/s$.
very · slow!

but the electrical force (field) transmits much faster!
(speed of light).

- e.g. 4-9. p. 87.
  conductance of Coaxial Cable.



Find G' (conductance per unit length).

Current I can "leak" through the insulating material ($\sigma \neq 0$).

Current density: $\vec{J} = \hat{r}\dfrac{I}{A} = \hat{r}\dfrac{I}{2\pi r \ell}$

Since $\vec{J} = \sigma \vec{E}$:

$$\vec{E} = \hat{r}\frac{I}{2\pi r \sigma \ell}$$

$$V_{ab} = V_a - V_b = -\int_b^a \vec{E}\cdot d\vec{\ell}$$

$$= -\int_b^a \frac{I\,dr}{2\pi\sigma\ell\cdot r} \qquad (d\vec{\ell} = d\vec{r})$$

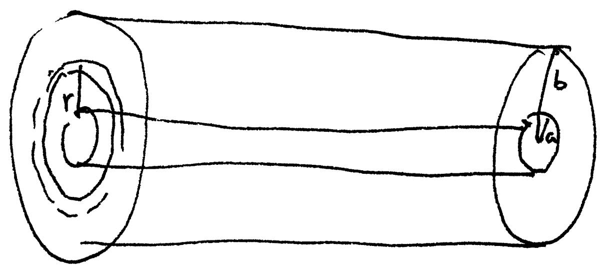
$$= \frac{I}{2\pi\sigma\ell}\ln\frac{b}{a}.$$

Conductance: $G = \dfrac{I}{V_{ab}} = \dfrac{2\pi\sigma\ell}{\ln\frac{b}{a}}.$

conductance per unit length:

$$G' = \frac{G}{\ell} = \frac{2\pi\sigma}{\ln b/a}.$$

OR: $dR = \rho \dfrac{dr}{2\pi r l} = \dfrac{dr}{2\pi \sigma l r}$ $\left(= \rho \dfrac{\text{length}}{\text{cross section area}} \right)$

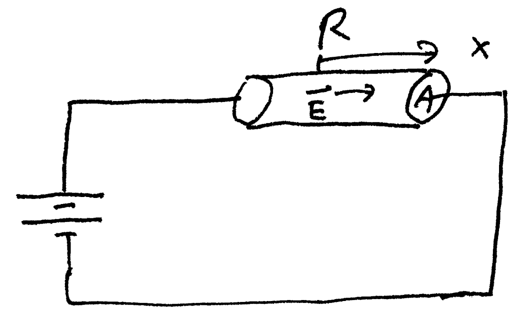


$$R = \int_a^b \frac{dr}{2\pi\sigma l r} = \frac{1}{2\pi\sigma l} \ln\frac{b}{a}$$

$$G = \frac{1}{R} = \frac{2\pi\sigma l}{\ln\frac{b}{a}}$$

$$G' = \frac{G}{l} = \frac{2\pi\sigma}{\ln\frac{b}{a}}$$

- Joule's law.



Power dissipated in R:

circuit: $P = IV = I^2 R$.

field: $P = \int_V \vec{E} \cdot \vec{J} \, dV = \int_V \sigma |\vec{E}|^2 dV$

verify: $\int \sigma |\vec{E}|^2 dV$

$= \sigma \int_A E_x \, dA \cdot \int_0^l E_x \, dx$ (Assuming uniform $\vec{E}$ along x-direction.)

$= \sigma E_x A \, E_x l$

$= J \cdot A \cdot E \cdot l$

$= I \cdot V$.