Phys 221

lecture 9.

Ref:
4-8, 4-9, 4-10, 4-11.

Rules for formula sheet.

① Hand written in ink.

② No diagram, graphs, figures.

③. No examples.

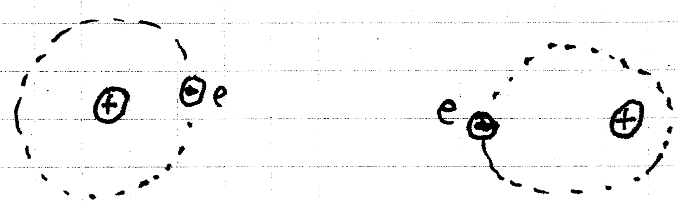
④ Textbook numbered equations only.

⑤. Max. 3 words per formula.

- Dielectrics.

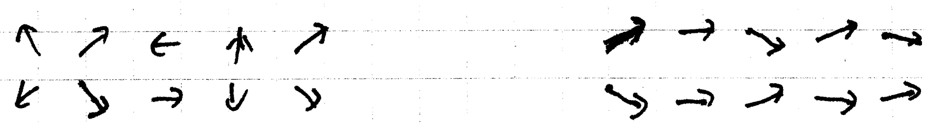
Effects of an applied $\vec{E}$-field on dielectric materials.

— induced dipoles. $\vec{E}_a = 0$ $\qquad$ $\vec{E}_a \neq 0$ $\xrightarrow{\vec{E}_a}$



— orientation of permanent dipoles (in polar materials)

$\vec{E}_a = 0$. $\qquad$ $\vec{E}_a \neq 0$ $\longrightarrow$



random.

- Polarization field: $\vec{P} = \dfrac{\Sigma \vec{p}}{\Delta V}$. (Total dipole per unit volume)

Note: lots of cancellation can occur in $\Sigma \vec{p}$ for dielectric materials. (See figure 4-17 on p.92).

- For linear, isotropic and homogeneous medium.

$$\vec{P} = \varepsilon_0 \chi_e \vec{E} \qquad \chi_e \text{ — electric susceptibility.}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

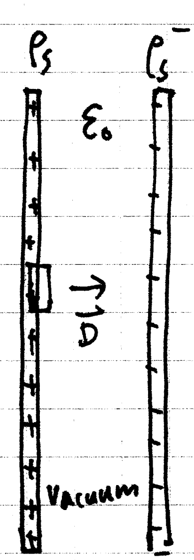
$$= \varepsilon_0 (1+\chi_e) \vec{E} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon \vec{E}$$

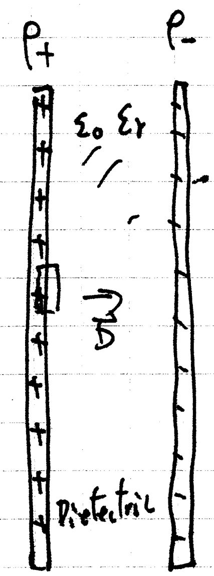
$$\varepsilon_r = 1+\chi_e \quad \text{— dielectric constant.}$$

$$= \frac{\varepsilon}{\varepsilon_0} \qquad \text{(relative electric permittivity).}$$

- Meaning of dielectric constant ε_r

e.g. Parallel plates with surface charge density ρ_s and ρ_s^-





$$\oint \vec{D} \cdot \vec{ds} = Q$$

$$DA = \rho_s A$$

$$D = \rho_s, \quad E = \frac{D}{\varepsilon_0} = \frac{\rho_s}{\varepsilon_0}.$$

$$\oint \vec{D} \cdot \vec{ds} = Q$$

$$D = \rho_s.$$

$$E = \frac{D}{\varepsilon_0 \varepsilon_r} = \frac{\rho_s}{\varepsilon_0 \varepsilon_r}.$$

$\therefore$ For the same charge (ρ_s, ρ_s^-),

$$E_{medium} = \frac{E_{vacuum}}{\varepsilon_r} \quad (\varepsilon_r > 1)$$

$$< E_{vacuum}.$$

$\vec{P}$ weakens the electric field in medium by a factor of ε_r.

$\uparrow$

(the induced dipoles can cancel (partly) the apply field.).

Typical value of ε_r:

air	$\varepsilon_r = 1.0006$	
polystyrene	2.6.	
Quartz	5.	

- Typical calculation involving dielectric medium:

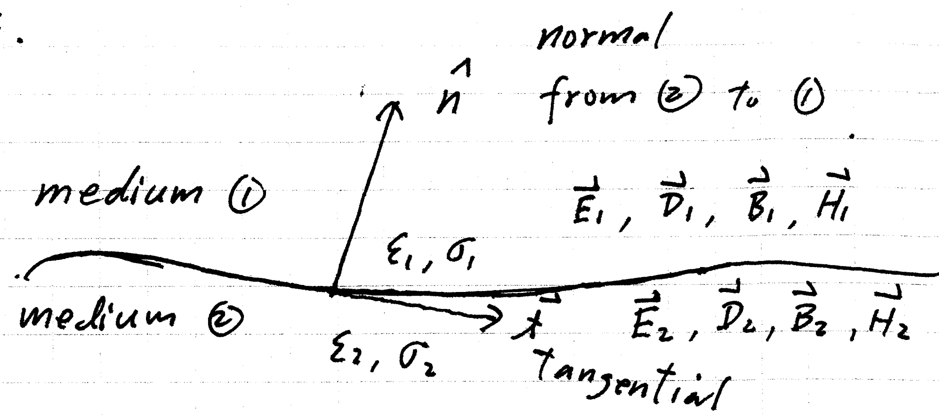
① $\vec{D}$ is related to free charges (through Gauss' law).

② . $\vec{E} = \dfrac{\vec{D}}{\varepsilon_0 \varepsilon_r} = \dfrac{\vec{D}}{\varepsilon}$.

③ . $\vec{P} = \vec{D} - \varepsilon_0 \vec{E}$

- Boundary conditions.

Typical situation



Difficulty: physical quantities $(\vec{E}, \vec{D}, \vec{B}, \vec{H}, \varepsilon, \sigma)$ are discontinuous at the boundary.

i.e, derivatives are undefined. $(\nabla, \nabla\cdot, \nabla\times, \text{etc.})$

Solution: Use the integral form of Maxwell's equations.
(All we need is that the physical quantities are finite, i.e, integrable).

Results. Table 4-3 on page 96.
↑ will NOT be provided in exams!

most important ones:

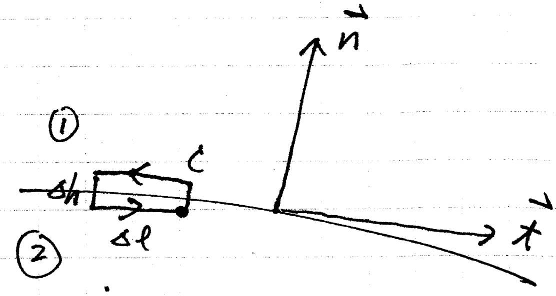
(4.90) $E_{1t} = E_{2t}$ (Tangential component of $\vec{E}$ is continuous)

(4.94) $D_{1n} - D_{2n} = \rho_s$ (ρ_s — free charge surface density)

i.e. $\hat{n}\cdot(\vec{D}_1 - \vec{D}_2) = \rho_s$

- Why $E_{1t} = E_{2t}$?

Because $\oint_C \vec{E} \cdot d\vec{l} = 0$



closed path C: $\Delta h \to 0$

$\Delta l \sim$ small.

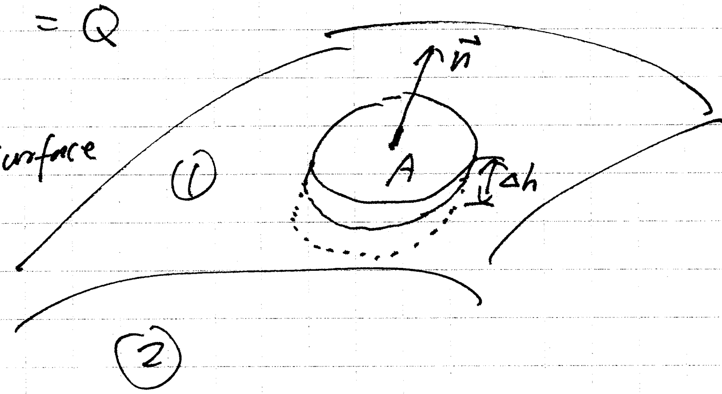
so that $\vec{E} \approx$ const. along Δl.

$$\oint_C \vec{E} \cdot d\vec{l} = E_{2t} \cdot \Delta l - E_{1t} \Delta l = 0$$

$$\therefore \quad E_{2t} = E_{1t} .$$

- why $D_{1n} - D_{2n} = \rho_s$?

because $\oint_S \vec{D} \cdot d\vec{s} = Q$



Small pill-like Gaussian surface

(very thin, $\Delta h \to 0$)

A — small so that $\vec{D}$ = const.

$$\oint_S \vec{D} \cdot d\vec{s} = D_{1n} A - D_{2n} A = \rho_s A .$$

$\Delta h \to 0$
D — finite.
$\therefore$ lateral flux $\to 0$

$$\therefore \quad D_{1n} - D_{2n} = \rho_s$$

Note: Definition of $\hat{n}$: from medium ② to medium ①.

Note: All the other boundary conditions in table 4-3 are just examples of

$$\begin{cases} E_{1t} = E_{2t} \\ D_{1n} - D_{2n} = \rho_s \end{cases}.$$

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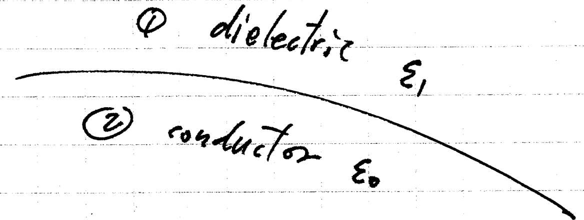
e.g $\begin{cases} ① \text{ — dielectric} \\ ② \text{ — conductor} \end{cases}$.



Since $\vec{E}_2 = 0$, $E_{2t} = 0$ $\quad D_{2t} = 0$ $\quad (\because \vec{D}_2 = \varepsilon_0 \vec{E}_2)$

$E_{1t} = 0$ $\quad D_{1t} = 0$ $\quad (\because \vec{D}_1 = \varepsilon_1 \vec{E}_1)$

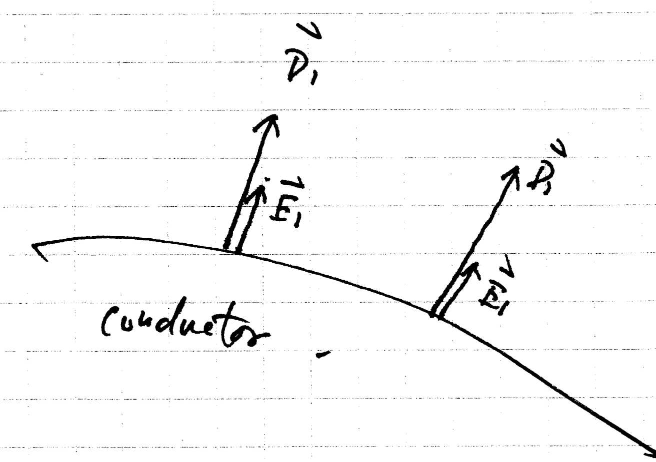
$(\because E_{1t} = E_{2t})$.

Also, since $\vec{E}_2 = 0$, $E_{2n} = 0$, $D_{2n} = 0$.

$$D_{1n} - D_{2n} = \rho_s$$

$$\therefore D_{1n} = \rho_s$$

$$E_{1n} = \frac{\rho_s}{\varepsilon_1}$$



$\therefore$ $\vec{D}$, $\vec{E}$ are perpendicular to the surface of conductor.

Assignment : 4.45.

good exercise for using B. Cs.

but, ~~is~~ there are conceptual problems in the solution! what are they?

① Maxwell's eqs.
② conservative.