Phys 221

Lecture 10.

Ref. 4-9, 4-10, 4-11.

- Ohmic media ($\sigma \neq 0$, but $\sigma \neq \infty$)

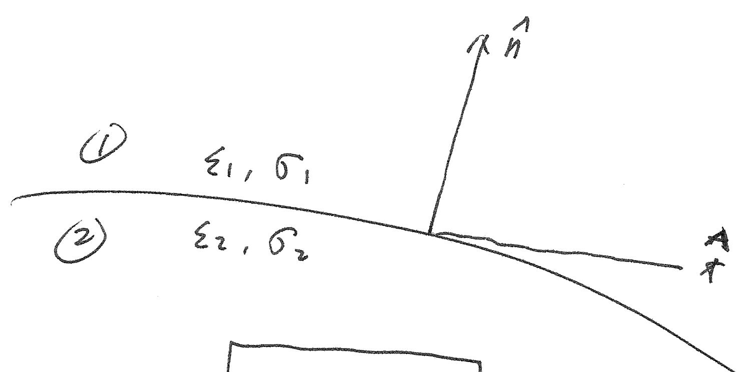
  conductor, but not perfect.

  current can flow, if $\vec{J} \neq 0$, $\vec{E} \neq 0$.

  (we can have a steady state, but not in static equilibrium.
  Charges can move, but at a constant rate, $\vec{J}$ = t-independent)

- ~~Conduc~~

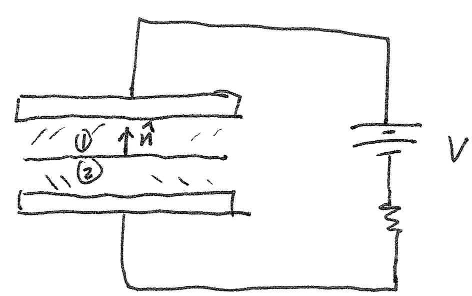
- conductor-conductor boundary



$$E_{1t} = E_{2t}$$

$$\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s .$$

e.g. capacitor



$$\vec{J} \neq 0, \quad \vec{J} = \sigma \vec{E}, \quad \vec{E} = \frac{\vec{J}}{\sigma}$$

$$\therefore \quad \frac{J_{1t}}{\sigma_1} = \frac{J_{2t}}{\sigma_2}$$

$$\varepsilon_1 \frac{J_{1n}}{\sigma_1} - \varepsilon_2 \frac{J_{2n}}{\sigma_2} = \rho_s .$$

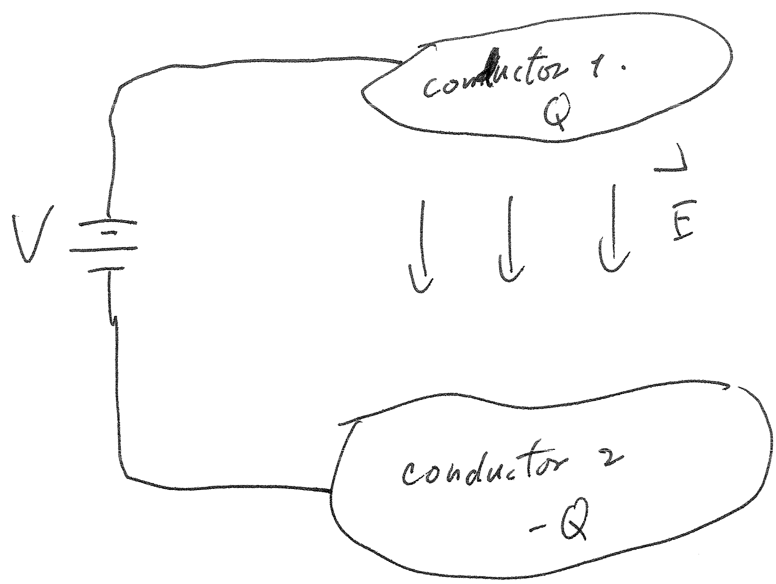
- Continuity condition for steady current.

$J_{1n} = J_{2n}$. — otherwise charges will accummulate at the boundary.
— impossible.

(i.e.
If $J_{1n} \neq J_{2n}$.
~~chag~~ charge in $\neq$ charge out at surface)

Then: $J_{1n} \left(\frac{\varepsilon_1}{\sigma_1} - \frac{\varepsilon_2}{\sigma_2} \right) = \rho_s$. (steady state)
$\vec{E}, \vec{D}, \vec{J}$ are all t-independent.

- Capacitance

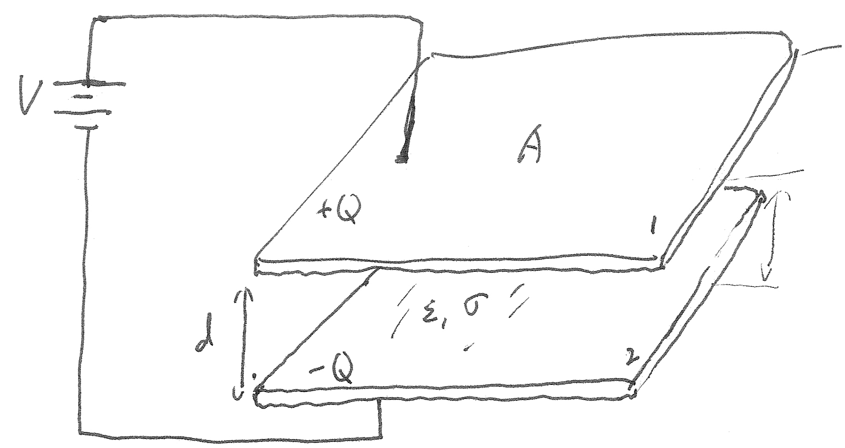


for a given configuration, the more the Q the higher the V

$C = \frac{Q}{V}$. The capacity of holding charge at the expense of voltage.
(charge per unit voltage).

e.g. Parallel plate capacitor

$C = ?$



connect V so that Q will build up (charging)

$$C = \frac{Q}{V} = \frac{\int_A \varepsilon \vec{E} \cdot d\vec{s}}{-\int_2^1 \vec{E} \cdot d\vec{l}} = \frac{\varepsilon E A}{E d} = \frac{\varepsilon A}{d}.$$

How to increase C: increase A.

decrease d.

increase ε. — use medium.

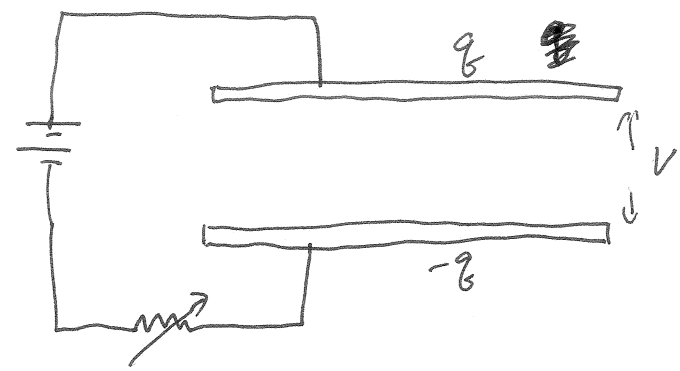
Resistance: $R = \frac{1}{\sigma} \frac{d}{A}$

OR: $R = \frac{V}{I} = \frac{\int_2^1 \vec{E} \cdot d\vec{l}}{\int_A \sigma \vec{E} \cdot d\vec{s}} = \frac{Ed}{\sigma E A} = \frac{d}{\sigma A}.$

$$\boxed{R \cdot C = \frac{\varepsilon}{\sigma}} \quad (4.111).$$

· Electric potential Energy stored in a capacitor.

$$W_e = \frac{1}{2}QV.$$
$$= \frac{1}{2}CV^2$$
$$= \frac{1}{2}\frac{Q^2}{C}.$$



Question: why not $W_e = QV$? $C = \frac{Q}{V}$

V — P.E. per unit charge!

where does the "$\frac{1}{2}$" come from?

Reason: for a given C, V varies with q. $\left(V = \frac{q}{C}\right)$

As we charge the capacitor, q increase from 0 to Q.

v also increase from 0 to V.

$$\therefore \quad W_e = \int_0^Q v\,dq = \int_0^Q \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C}. = \frac{1}{2}QV = \frac{1}{2}CQ^2.$$
$$= \frac{1}{2}CV^2$$

· electric field: $E \cdot d = V$, $E = \frac{V}{d}$

$$W_e = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon A}{d}\cdot E^2 d^2 = \frac{1}{2}\varepsilon \underbrace{Ad}_{\text{volume}} \cdot E^2.$$

Electric field energy density:

$$\boxed{w_e = \frac{1}{2}\varepsilon E^2}$$ 4-123.