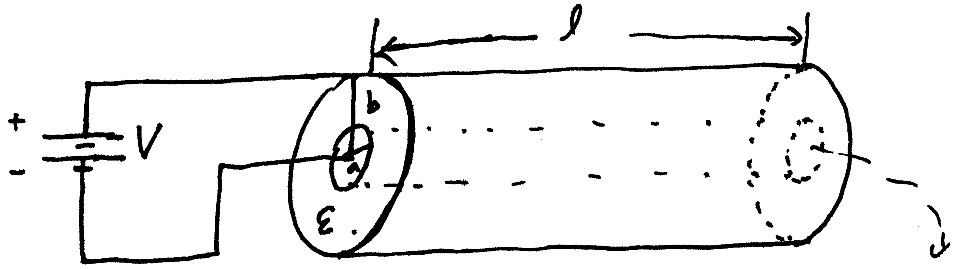
Phys 221.

Lecture 11.

Ref: 4-10, 4-11, 4-12; 5-1.

e.g. Capacitance of Coaxial line



charge per unit length: ρ_l

in ε: $\vec{E} = -\hat{r}\frac{\rho_l}{2\pi\varepsilon r}$ (Using Gauss' law)

$$= -\hat{r}\frac{Q}{2\pi\varepsilon r\cdot l}$$

$$V = -\int_a^b \vec{E}\cdot d\vec{r} = \frac{Q}{2\pi\varepsilon l}\int_a^b \frac{dr}{r} = \frac{Q}{2\pi\varepsilon l}\ln\frac{b}{a}.$$

$$C = \frac{Q}{V} = \frac{2\pi\varepsilon l}{\ln(b/a)},$$ per unit length: $C' = \frac{2\pi\varepsilon}{\ln(\frac{b}{a})}$

Resistance: $$R = \frac{1}{\sigma}\int_a^b \frac{dr}{2\pi r l} = \frac{1}{2\pi\sigma l}\ln\frac{b}{a}.$$

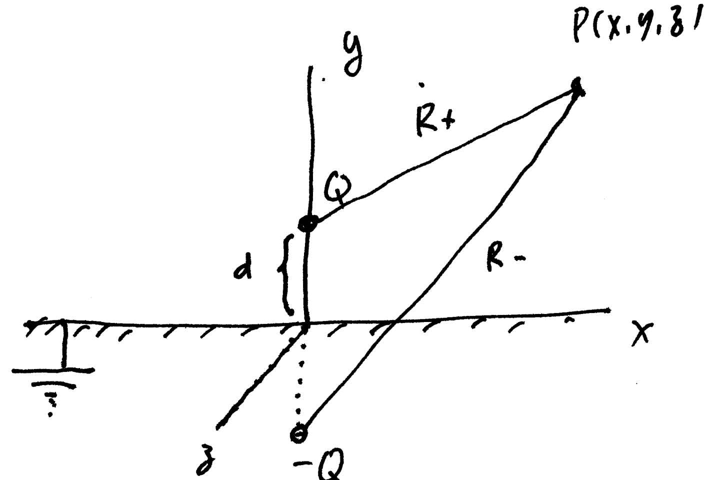
$RC = \frac{\varepsilon}{\sigma}$. — same as parallel-plate capacitor.

· Image Method .

— A very clever way to solve certain electrostatic problems .

e.g. A point charge Q and a grounded plane conductor .

find $V(x, y, z)$ for $y > 0$



Difficulty: unknown charge distribution on the conductor surface . $\rho_s \neq 0$.

Idea: Use an equivalent charge ("image charge") to replace the conductor .

This "equivalent" problem has the same equations

$$\nabla^2 V = \rho/\varepsilon_0 \text{ in } y > 0 .$$

and the same boundary conditions $V = 0$ at $y = 0$.

We don't care if it's equivalent in $y < 0$, because we know that $V = 0$ in $y < 0$.

The image charges do not change the charge distributions in the area of interest, They ensure the boundary conditions is satisfied on the conductor surfaces .

$$\therefore \ V(x, y, z) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R_+} + \frac{-Q}{R_-}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2 + (y-d)^2 + z^2}} - \frac{1}{\sqrt{x^2 + (y+d)^2 + z^2}}\right)$$

To find $\vec{E}$: $\vec{E} = -\nabla V$. OR use the textbook approach. (p.106) .

To find . ρ_s on conductor : exercise 4.19. p.107 .

- Magnetostatics
  - t-independent Maxwell's equations on magnetism:

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J} \quad . \qquad \left(\frac{\partial \vec{D}}{\partial t} = 0 \, , \right.$$

($\vec{J} \neq 0$, but $\vec{J}$ – steady. ($\frac{\partial \vec{J}}{\partial t} = 0$))

charge can move!

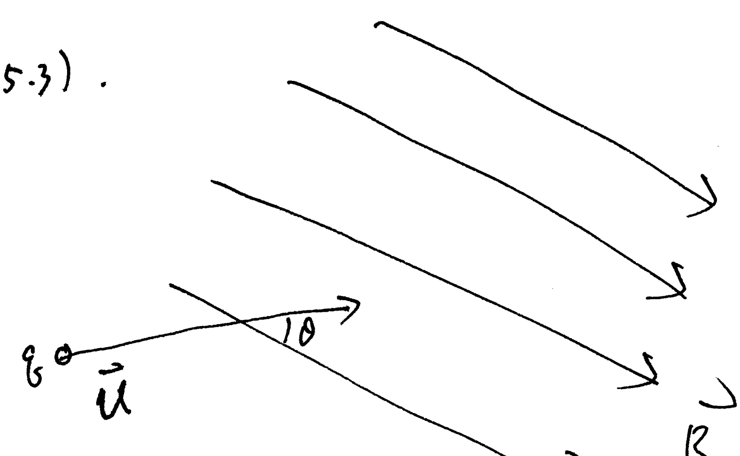
  - Material : μ (magnetic permeability)

$$\vec{B} = \mu \vec{H} \qquad \text{(except ferromagnetic materials)}$$

- Magnetic force on a moving charge:

$$\vec{F}_m = q \, \vec{u} \times \vec{B} \qquad \vec{u} \text{ – velocity} .$$

(5.3) .



(5.3) can be considered as the definition of $\vec{B}$.

RHR : $\vec{F}_m$: $\otimes$ (into page)

$$F_m = q u B \sin\theta \, .$$

$$= q u_\perp B$$

$$= q u B_\perp$$

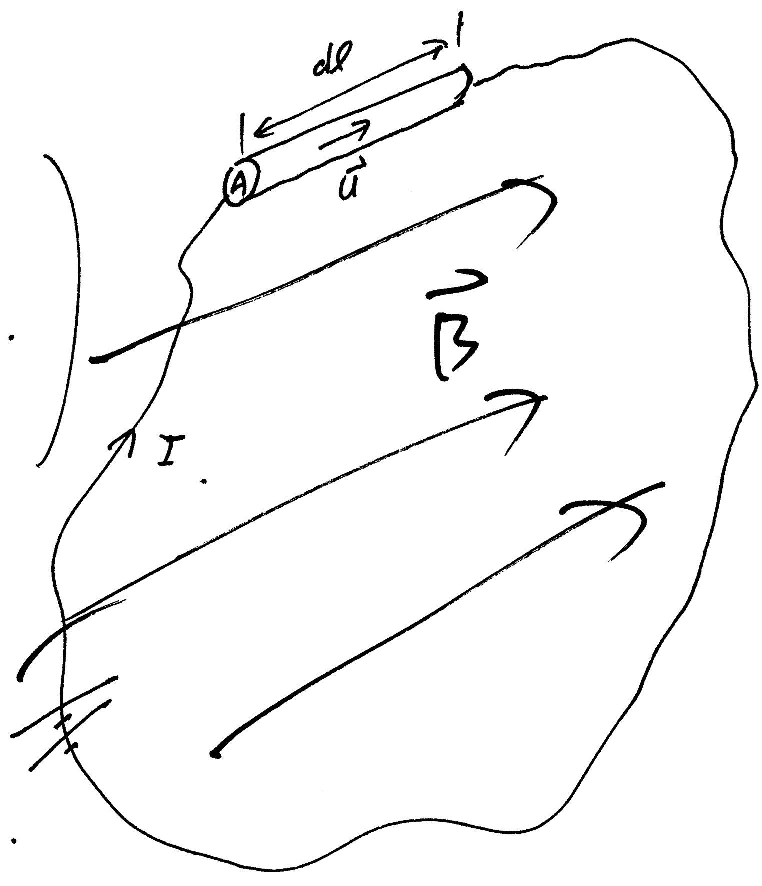
- Lorentz force

  electromagnetic force on a moving charge q.

$$\vec{F} = q(\underbrace{\vec{E}}_{\text{electric force}} + \underbrace{\vec{u} \times \vec{B}}_{\text{magnetic force}})$$

- Magnetic force on a current element:

$$d\vec{F}_m = I\,d\vec{l} \times \vec{B}.$$



(Note: $I\,d\vec{l} = Nq\vec{u}$

$$= Nq\frac{d\vec{l}}{dt}.$$

$$\frac{Nq}{dt} = I.$$)

Note: $I\,d\vec{l}$ is an element of a current I.

It can not exist without a ~~c~~ closed current.

Total magnetic force on the closed circuit:

$$\vec{F}_m = I \cdot \oint_C d\vec{l} \times \vec{B} \qquad 5.10.$$