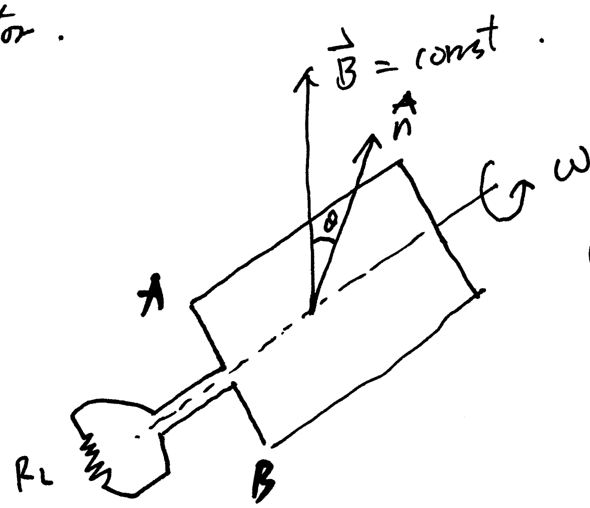
Phys 221.

Lecture 19.

Ref: 6-5, 6-6, 6-7*
6-8, 6-9, 6-10.

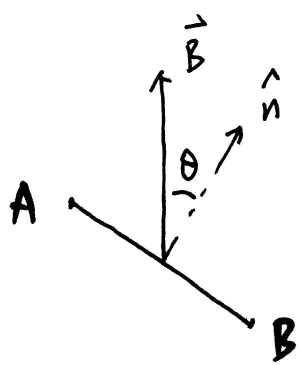
- The electromagnetic generator.

fig. 6-12. (p.181)



$\theta = \omega t$

front view.



flux $\Phi = \int \vec{B} \cdot d\vec{s} = BA \cdot \cos\theta$

$\Phi(t) = BA \cos(\omega t)$

$V_{emf} = -\frac{d\Phi}{dt} = BA\omega \sin(\omega t)$.

Alternating current.

what if $\vec{B}$ is also t-dependent?

$\vec{B} = \hat{z} B_0 \cos(\omega t)$

Then: $\Phi(t) = B_0 A \cos^2(\omega t)$.

$V_{emf} = -\frac{d\Phi}{dt} = 2 B_0 A \cos(\omega t) \sin(\omega t) \cdot \omega$.

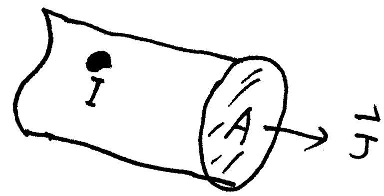
$= B_0 \omega A \sin(2\omega t)$. $\quad (\sin 2x = 2 \sin x \cos x)$

- charge-current continuity relation

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t} \qquad 6.54 \ .$$

What does it mean?

$$I = \int_A \vec{J} \cdot d\vec{s}$$



$$= \frac{Q}{\delta t} = \frac{\text{amount of charge through A}}{\text{per unit time}} \ .$$

Total current going out of a closed surface S:

$$\oint_S \vec{J} \cdot d\vec{s} = -\frac{dQ}{dt} \ . \quad \left(\begin{array}{c} \text{decreasing in } Q \\ \text{per unit time} \\ \text{in } V \end{array} \right)$$

$$= -\frac{d}{dt} \int_V \rho_v \, dV$$

Q — charge in V

(V — enclosed volume)

$$\int_V (\nabla \cdot \vec{J}) \, dV = -\int_V \frac{\partial \rho_v}{\partial t} \, dV \ .$$

This is true for any volume V.

$$\therefore \quad \nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t} \ . \qquad \text{(charge is the source of current)}$$

OR: $$\nabla \cdot \vec{J} + \frac{\partial \rho_v}{\partial t} = 0 \ . \qquad \text{(conservation of electric charge).}$$

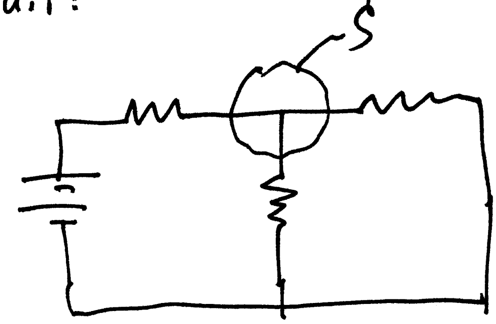
e.g. In a dc circuit, $\frac{\partial \rho_v}{\partial t} = 0$.

$$\nabla \cdot \vec{J} = 0 .$$

i.e. $\oint_S \vec{J} \cdot d\vec{s} = 0$ for any closed S.

in a node of a DC circuit:

$$\sum I_{out} = 0 .$$



That's Kirchoff's current law.