Phys 221

Lecture 27.

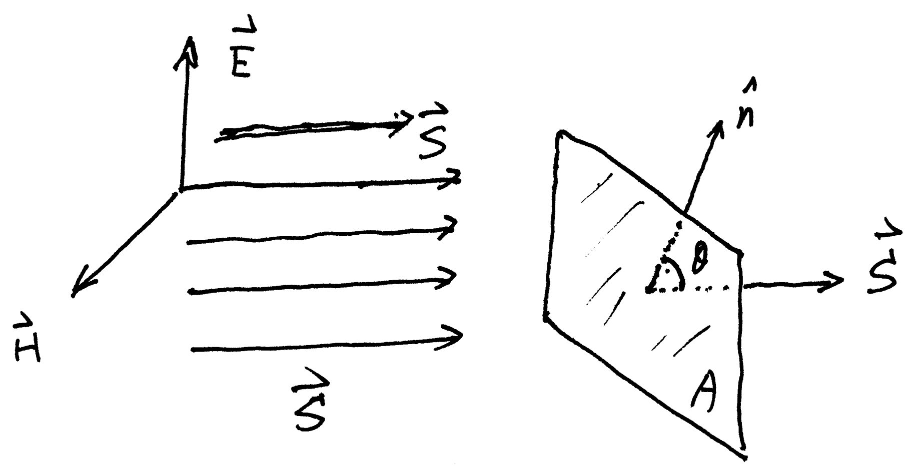
Ref. 7-7. Electromagnetic Power density.

- Poynting vector

$$\vec{S} = \vec{E} \times \vec{H}$$

Meaning : Direction : propagation direction ($\hat{S} = \hat{k}$)

Magnitude : Power through per unit area.



Power through area A :

$$P = \int_A \vec{S} \cdot \hat{n} \, dA = \int_A \vec{S} \cdot d\vec{A}$$

- In a lossless medium, e.g. plane wave travelling in $\hat{z}$ direction

— Time domain: $\vec{S}(z,t) = \vec{E}(z,t) \times \vec{H}(z,t)$

(Instantaneous relation) $= \hat{z} \dfrac{|E_{x0}|^2}{\eta} \cos^2(\omega t - kz)$ $(k = \beta)$

time-average (over a period T):

$$\vec{S}_{av} = \frac{1}{T}\int_0^T \vec{S}(z,t)\,dt = \hat{z}\frac{|E_{x0}|^2}{2\eta} \quad (7.157)$$

$$\left(\because \frac{1}{2\pi}\int_0^{2\pi} \cos^2 x\,dx = \frac{1}{2} \right)$$

— phasor domain: $\tilde{\vec{S}} \neq \tilde{\vec{E}} \times \tilde{\vec{H}}$ (because it's not a linear operation. X)

It turns out:
(because of 7.157) $\boxed{\vec{S}_{av} = \frac{1}{2} \mathrm{Re}\left[\tilde{\vec{E}} \times \tilde{\vec{H}}^* \right]}$ (7.158)

$\tilde{\vec{H}}^*$ — complex conjugate of $\tilde{\vec{H}}$

replace j with $-j$.

Verify 7.158: $\frac{1}{2}\mathrm{Re}\left[\tilde{\vec{E}} \times \tilde{\vec{H}}^* \right] = \frac{1}{2}\mathrm{Re}\left[\hat{x} E_{x0} e^{-jkz} \times \hat{y} \frac{E_{x0}^*}{\eta} e^{jkz} \right]$

$= \frac{1}{2}\mathrm{Re}\left[\hat{z} \frac{|E_{x0}|^2}{\eta} \right] = \frac{1}{2} \hat{z} \frac{|E_{x0}|^2}{\eta}.$

- In a lossy medium.

$$\eta_c = |\eta_c| e^{j\theta_\eta}$$

Then, can be shown:

$$\vec{S}_{av}(z) = \hat{z} \frac{|E_0|^2}{2 \cdot |\eta_c|} e^{-2\alpha z} \cos(\theta_\eta)$$

at $z = \delta_s$ (skin depth).

$$E = E_0 e^{-1} = 37\% E_0$$

$$S_{av} = S_{av}(0) \cdot e^{-2} = 14\% S(0).$$

- Decibel Scale.

  - For power ratio $G = \frac{P_1}{P_2}$.

$$G[dB] = 10 \log \frac{P_1}{P_2}.$$

  - For voltage, current, field $\vec{E}$, $\vec{H}$, $g = \frac{V_1}{V_2}$

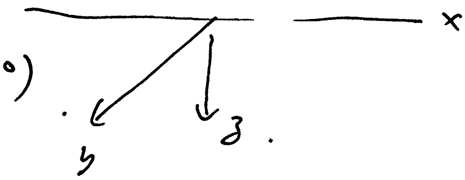
$$g[dB] = 20 \log \frac{V_1}{V_2}.$$

(why? $\because P = \frac{V^2}{R}$, so G and g in [dB]
$P \propto I^2, E^2, H^2$, etc. have the same meaning)

e.g. 7-10. A plane wave travelling in ~~the~~ sea water.

$$\vec{E}(z,t) = \hat{x}\, 4.4\, e^{-0.126z} \cos(2\pi \times 10^3 t - 0.126z + 60^\circ)$$

$$\vec{H}(z,t) = \hat{y}\, 100\, e^{-0.126z} \cos(2\pi \times 10^3 t - 0.126z + 15^\circ)$$



A submarine at a depth of 200 m uses a wire antenna to receive signal transmission at 1 KHz. Determine the averge power density incident upon the submarine antenna.

$|E_0| = 4.44$ mV/m

$\alpha = 0.126$

$\eta_c = 0.044 \angle 45^\circ$

$$\vec{S}_{av}(z) = \frac{1}{2} Re\left[\tilde{\vec{E}} \times \tilde{\vec{H}}\right]$$

$$= \hat{z} \cdot \frac{|E_0|^2}{2|\eta_c|} e^{-2\alpha z} \cos\theta_\eta$$

$$= \hat{z} \frac{(4.44 \times 10^{-3})^2}{2 \times 0.044} e^{-0.252z} \cos 45^\circ$$

$$= \hat{z}\, 0.16\, e^{-0.252z} \quad (\mathrm{mW/m^2})$$

at $z = 200$ m.

$$\vec{S}_{av} = \hat{z}\, 0.16 \times 10^{-3}\, e^{-0.252 \times 200}$$

$$= 2.1 \times 10^{-26}\ \mathrm{W/m^2}$$