

Phys 221.

Lecture 28.

Ref. 7-7, 8-1, 8-2, 8-3, 8-4.

• Poynting vector: $\vec{S} = \vec{E} \times \vec{H}$ (Power density of E & M Fields)

For harmonic fields,

$$\vec{S}_{av} = \frac{1}{2} \text{Re} [\tilde{\vec{E}} \times \tilde{\vec{H}}]$$

• e.g. A long conducting wire of radius a and conductivity σ carries a DC current I .

at surface: $\vec{J} = \sigma \vec{E} = \frac{I}{\pi a^2} \hat{\delta}$

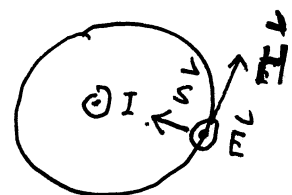
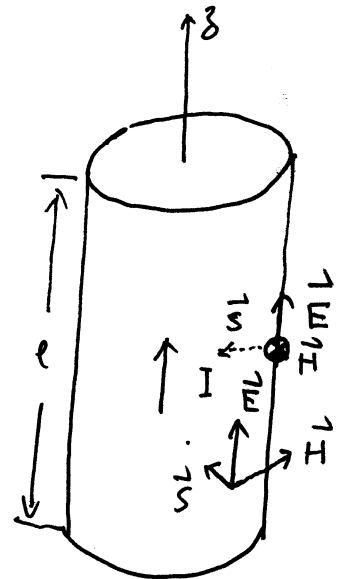
$$\vec{E} = \hat{\delta} \frac{I}{\pi a^2 \sigma}$$

$$\vec{H} = \hat{\phi} \frac{I}{2\pi a} \quad (\text{Ampere's law})$$

$$\vec{S} = \vec{E} \times \vec{H}$$

$$= -\hat{r} \frac{I^2}{2\pi^2 a^3 \sigma}$$

electromagnetic field energy flows across the surface into the wire.



Field energy flowing in to wire of length l , per unit time:

$$P_{\text{field}} = - \int \vec{S} \cdot d\vec{A}$$

$$= \frac{I^2}{2\pi^2 a^3 \sigma} \cdot 2\pi a \cdot l$$

$$= \frac{I^2 l}{\pi a^2 \sigma}$$

but: $\frac{1}{\sigma} \frac{l}{\pi a^2} = R$

$$\therefore P_{\text{field}} = \underbrace{I^2 R}_{\text{Joule's law}} = P_{\text{heat}}$$

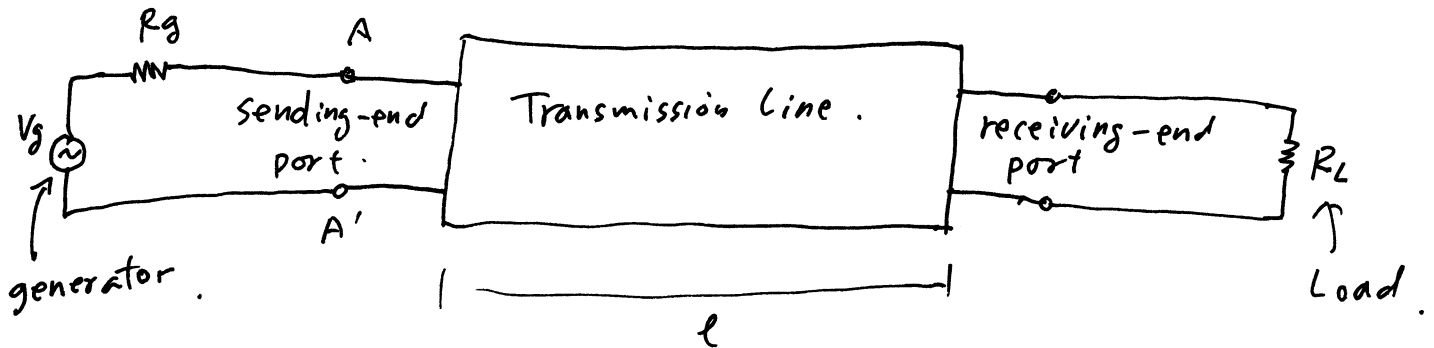
Joule's law.

i.e. ~~The ohmic~~ The ohmic power loss $I^2 R$ in the wire equals the power of EDM Field entering the wire.

$$\left(\begin{array}{l} \text{energy conservation} \\ \text{energy in} = \text{energy used} \end{array} \right)$$

• Transmission Lines .

A transmission line — Two-port network .



e.g.s. figure 8-4 on page 248 .

Two-wire line

Coaxial line

parallel-plate line .

typically
2-conductor lines.

etc .

In this course, we deal with TEM-mode lines only .

- For low-frequencies , the transmission line doesn't play an important role, and thus can be treated as a short .

However , for high frequencies , $\frac{l}{\lambda} > 1\%$ of λ ,

We need to consider the effects of the line .

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Why? Because it takes time for the E & M wave to travel through. $\hookrightarrow$ leads to a phase shift.
(for high-freq. signals)

e.g. at AA' $V_{AA'}(t) = V_0 \cos(\omega t)$

at BB' $V_{BB'}(t) = V_0 \cos[\omega(t - \frac{l}{c})]$

phase difference: $\Delta\phi = \frac{\omega l}{c} = \frac{2\pi}{T} \frac{l}{c} = 2\pi \frac{l}{\lambda}$

$\Delta\phi$ is negligible only if $\Delta\phi \ll 2\pi$.

i.e. $\frac{l}{\lambda} \ll 1$, or $l \ll \lambda$.

e.g. If $f = 1 \text{ kHz}$, $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10^3} = 3 \times 10^5 \text{ m} = 300 \text{ km}$.

for a short line of $l = 5 \text{ cm} = 0.05 \text{ m}$, $\ll \lambda$.

~~$V_{BB'}(t)$~~ phase shift $\frac{l}{\lambda} \cdot 2\pi \sim 1 \times 10^{-6} \text{ rad}$.

$V_{AA'}(0) = V_0$, $V_{BB'}(0) = 0.999999999998 V_0$

for a long line of $l = 20 \text{ km} = 2 \times 10^4 \text{ m}$.

$\frac{l}{\lambda} = 0.067$.

$V_{BB'}(0) = 0.91 V_0$ — the effects of transmission line needs to be analyzed.

Besides phase shift, there are other effects of transmission lines.

* — reflection of E & M signals.

— Power loss

— dispersion — wave velocity depends on freq.

$$(u \sim \frac{1}{\sqrt{\epsilon\mu}}, \text{ e.g. } \epsilon = \epsilon(\omega))$$

— leads to distortion of signals.

Note: A "long line" doesn't have to be long!

— relative to λ .

e.g: at $f = 10 \text{ GHz}$, $\lambda = 3 \text{ cm} = 0.03 \text{ m}$.

a line of 1 mm , $\frac{l}{\lambda} \approx 0.033$ (considered as "long")

even though
its length is
only 1 mm .

This can happen in an IC chip.