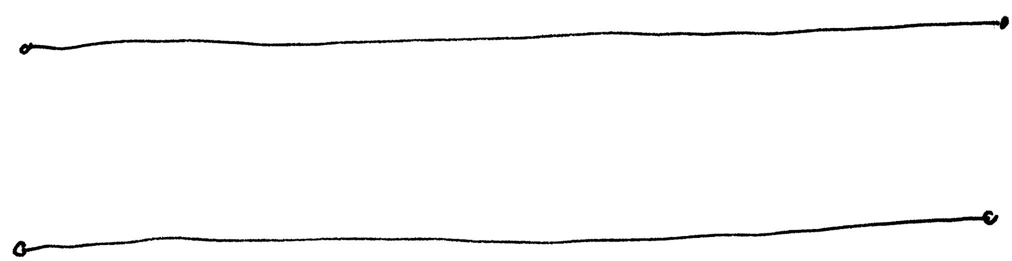
Phys 221

Lecture 29.

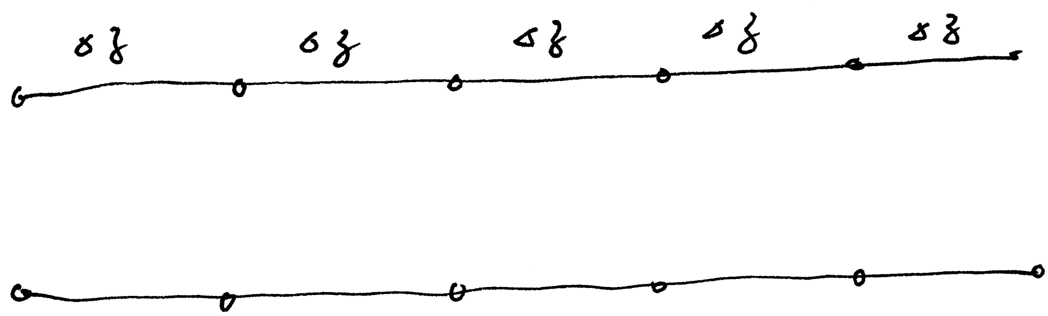
Ref. 8-1, 2, 3, 4.

- How do we analyze a transmission line. (Lumped-Element Model)

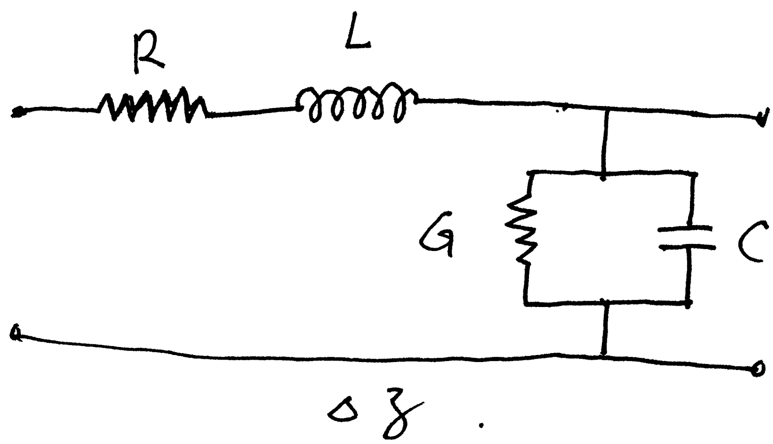


Divide it into many differential sections, each Δz long.

$\Delta z \to 0$



for each section, use an equivalent circuit.



$R = R'\Delta z$

$L = L'\Delta z$

$G = G'\Delta z$

$C = C'\Delta z$.

- Parameters representing the properties of the line

R' — Combined resistance of both conductors per unit length. (Ω/m)

L' — Inductance per unit length H/m.

G' — Conductance of the insulating medium per unit length. S/m

C' — capacitance per unit length F/m.

Results of R', L', G', C'

for coaxial, two-wire, parallel-plate lines

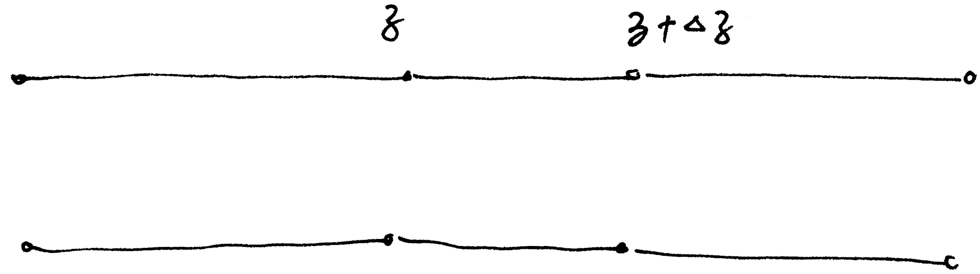
are listed in Table 8-1 on page 251.

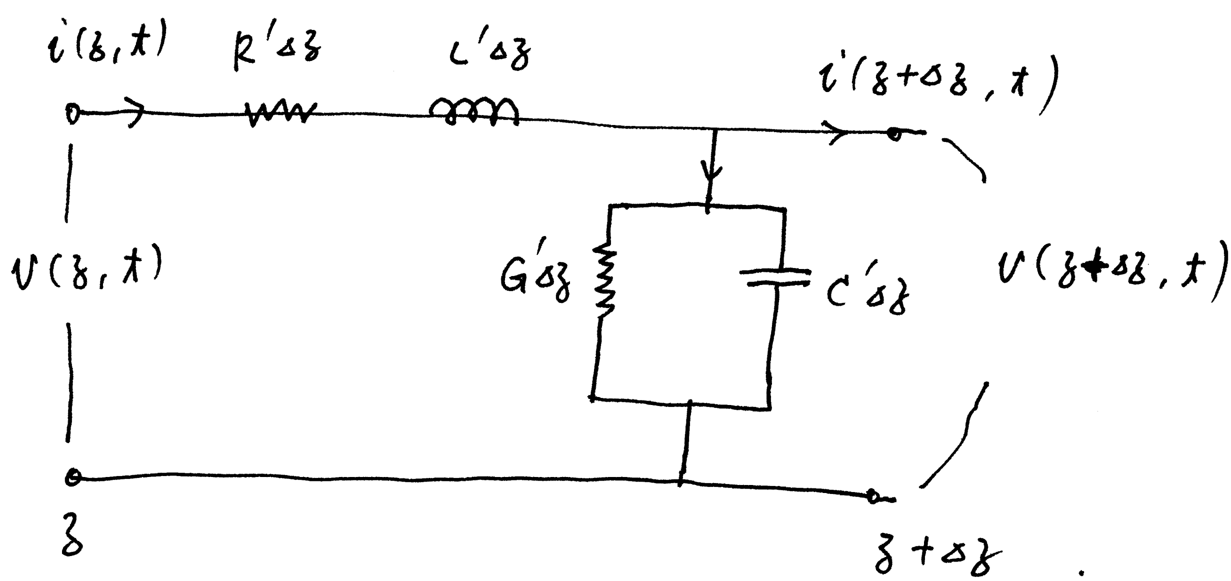
They depend on ① shape and dimensions

② material properties, (μ, ε, σ)

③ frequency.

- Transmission line equations
  (for $v(z,t)$ and $i(z,t)$)





.

KVL:

$$v(z,t) - R'\Delta z\, i(z,t) - L'\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z, t) = 0$$

$$-\left[\frac{v(z+\Delta z,t) - v(z,t)}{\Delta z}\right] = R' i(z,t) + L'\frac{\partial i(z,t)}{\partial t}.$$

$\Delta z \to 0$.

$$-\frac{\partial v(z,t)}{\partial z} = R' i(z,t) + L'\frac{\partial i(z,t)}{\partial t}. \qquad (8.14)$$

$C = \frac{q}{V}$

$q = CV$

$i = \frac{\partial q}{\partial t} = C\frac{\partial V}{\partial t}$.

KCL.

$i = VG$

$$i(z,t) - G'\Delta z \cdot v(z+\Delta z,t) - C'\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

$$-\left[\frac{i(z+\Delta z,t) - i(z,t)}{\Delta z}\right] = G' v(z+\Delta z,t) + C'\frac{\partial v(z+\Delta z,t)}{\partial t}.$$

$\Delta z \to 0$

$$-\frac{\partial i(z,t)}{\partial z} = G' v(z,t) + C'\frac{\partial v(z,t)}{\partial t}. \qquad (8.16)$$

- phasor domain:

$$\boxed{\begin{aligned} -\frac{d\tilde{V}(z)}{dz} &= (R' + j\omega L)\,\tilde{I}(z) \\ -\frac{d\tilde{I}(z)}{dz} &= (G' + j\omega C)\,\tilde{V}(z) \end{aligned}}$$

(8.18 a)

(8.18 b) 2×2 system of First-order ODEs

2 - unknow functions