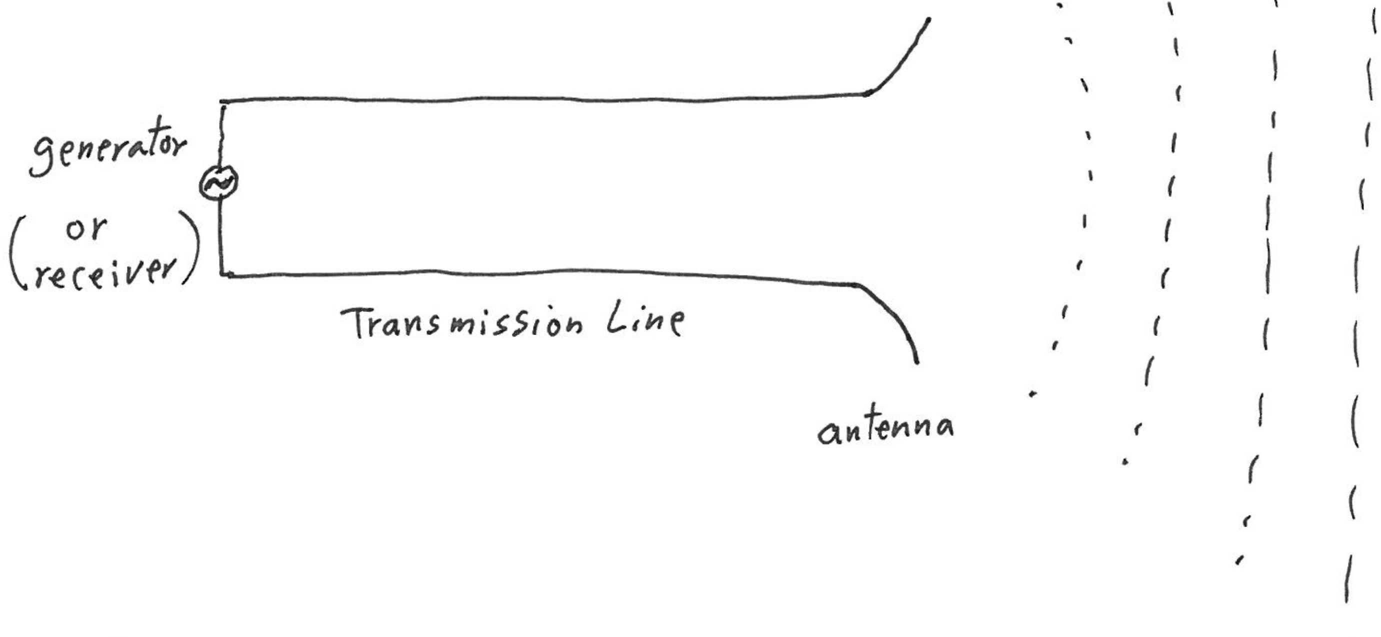
Phys 221.

Lecture 31, 32.

Ref. 10-1, 2. Radiation and Antennas

- Antenna — A transducer between a guided wave in a transmission line and an electromagnetic wave propagating in an unbounded medium or free space.



- Can be sending or receiving electomagnetic signals.

- Reciprocity. — same pattern for sending and receiving.

easier to analyze.

- 2 important aspects of antenna performance

  ① Radiation properties

  — Directional pattern

  — Polarization state

  ② Antenna impedance.

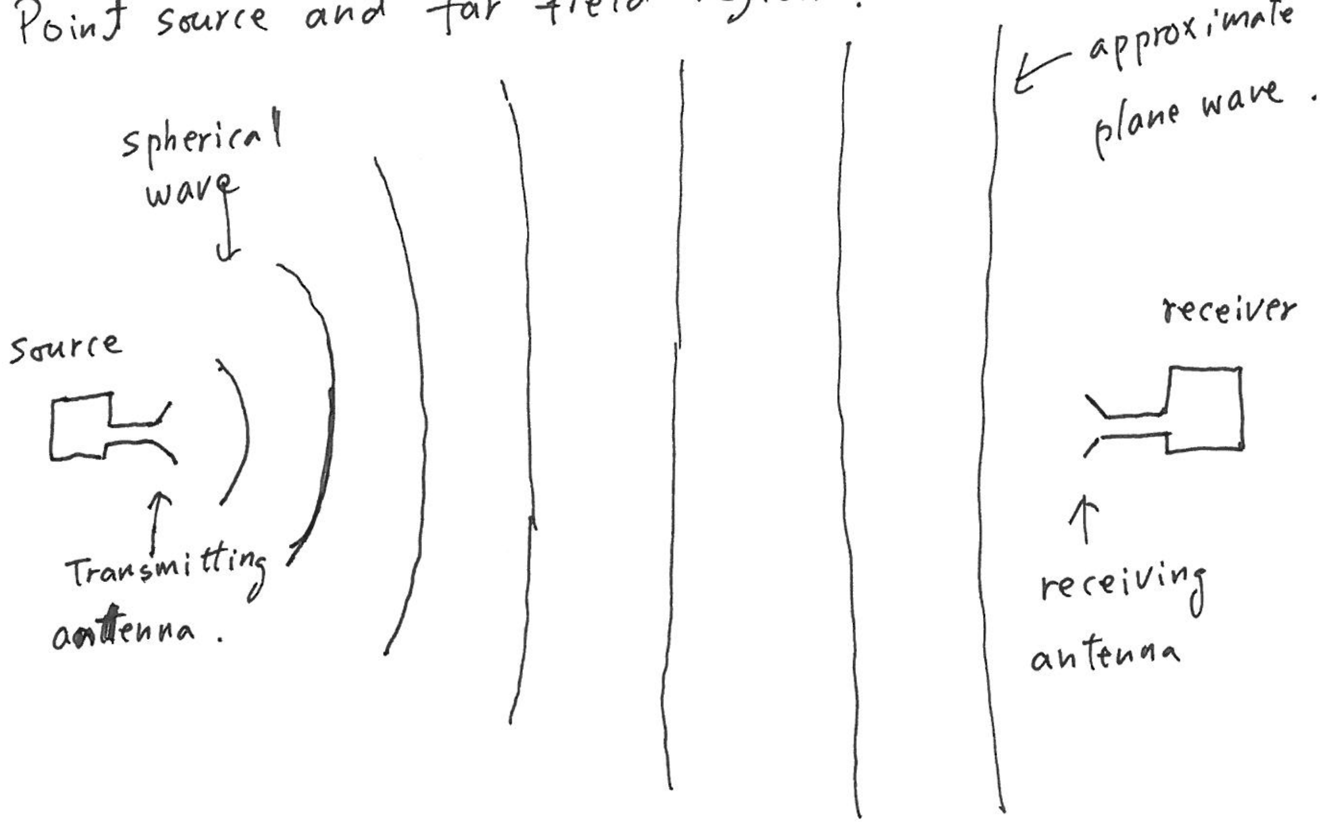
  — impedance matching

  — efficiency, loss, power etc.

- Radiation sources.

  time-varying current (moving charge).

- Point source and far field region.



- How to analyze.

$$\text{current } i(t) \Rightarrow \vec{A} \Rightarrow \vec{H} \Rightarrow \vec{E}$$

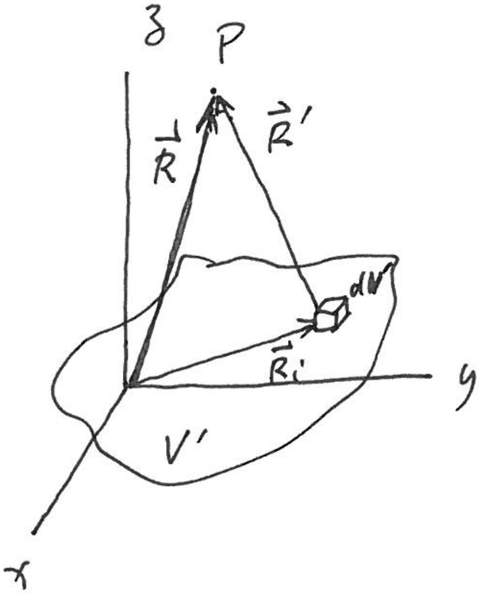
- Retarded potential.

$$\vec{A} = \frac{\mu}{4\pi}\int_{V'} \frac{\vec{J}(\vec{R}_i, t-\frac{R'}{u_p})}{R'}\, dV'$$

$[\vec{A}(\vec{R}, t)]$

$\vec{R}' = \vec{R} - \vec{R}_i$,

from source to field point.



u_p — velocity

It takes time $\frac{R'}{u_p}$ for the E & M action to reach P from dV'

- For harmonic time-dependence, $e^{j\omega t}$.

use phasor:

$$\tilde{\vec{A}}(\vec{R}) = \frac{\mu}{4\pi}\int_{V'} \frac{\tilde{\vec{J}}(\vec{R}_i)e^{-jkR'}}{R'}\, dV'$$

$k = \frac{\omega}{u_p} = \frac{2\pi}{\lambda}$ — wave number.

Then: $\tilde{\vec{H}} = \frac{1}{\mu}\nabla \times \tilde{\vec{A}}$, $\tilde{\vec{E}} = \frac{1}{j\omega\varepsilon}\nabla \times \tilde{\vec{H}}$ (propagation in free space)

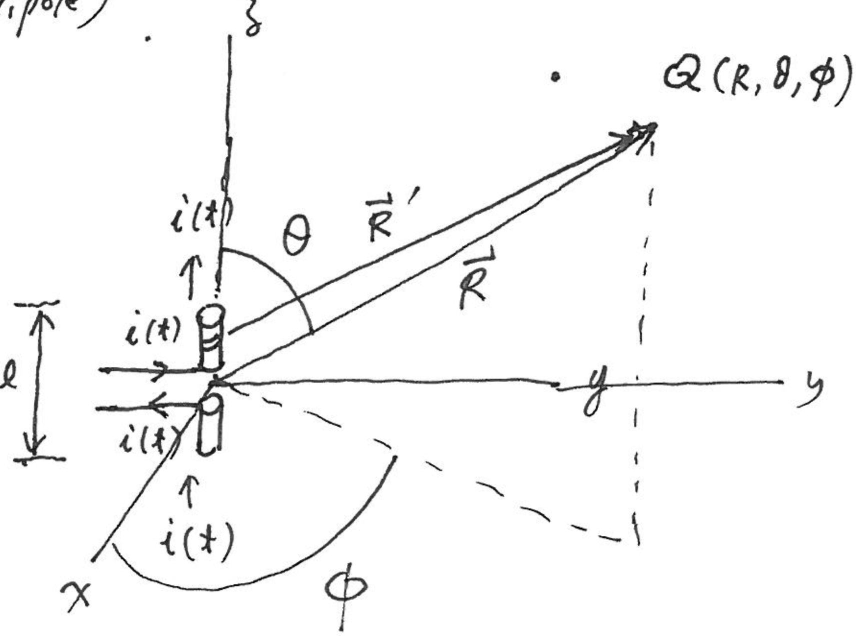
- The short dipole (Hertzian dipole).

$\frac{\ell}{\pi} < 50$

So that

$i(t)$ — ind. of z.

$R' \approx R$.



- Spherical coordinates.

R — Range

θ — zenith angle

ϕ — azimuth angle.

$i(t) = I_0 \cos(\omega t)$

$= Re\left[I_0 e^{j\omega t} \right]$

$\tilde{I} = I_0$

$$\vec{\tilde{A}}(\vec{R}) = \frac{\mu_0}{4\pi} \int_{V'} \frac{\vec{\tilde{J}} e^{-jkR'}}{R'} dV'$$

$$= \frac{\mu_0}{4\pi} \frac{e^{-jkR}}{R} \int_{-\ell/2}^{\ell/2} \hat{z} I_0 dz$$

$$= \hat{z} \frac{\mu_0}{4\pi} I_0 \ell \underbrace{\frac{e^{-jkR}}{R}}_{\text{spherical propagation factor}}$$

$$= \left(\hat{R}\cos\theta - \hat{\theta}\sin\theta \right) \frac{\mu_0 I_0 \ell}{4\pi} \frac{e^{-jkR}}{R}$$

then, $\vec{\tilde{H}} = \frac{1}{\mu_0} \nabla \times \vec{\tilde{A}}$, $\vec{\tilde{E}} = \frac{1}{j\omega\varepsilon} \nabla \times \vec{\tilde{H}}$

- Result : $H_R = 0$, $H_\theta = 0$, $E_\phi = 0$.

$$\tilde{H}_\phi = \frac{I_0 l k^2}{4\pi} e^{-jkR} \left[\frac{j}{kR} + \frac{1}{(kR)^2} \right] \sin\theta$$

$$\tilde{E}_R = \frac{2 I_0 l k^2}{4\pi} \eta_0 e^{-jkR} \left[\frac{1}{(kR)^2} - \frac{j}{(kR)^3} \right] \cos\theta$$

$$\tilde{E}_\theta = \frac{I_0 l k^2}{4\pi} \eta_0 e^{-jkR} \left[\frac{j}{kR} + \frac{1}{(kR)^2} - \frac{j}{(kR)^3} \right] \sin\theta .$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 120\pi$$

- Far field. $kR \gg 1$, $\left(\frac{2\pi}{\lambda} R \gg 1 \Rightarrow R \gg \lambda \right)$

$\frac{1}{(kR)^2}$, $\frac{1}{(kR)^3}$, die out quickly.

$$\tilde{E}_\theta = j \frac{I_0 l k \eta_0}{4\pi} \frac{e^{-jkR}}{R} \sin\theta \quad \propto \frac{1}{R}$$

$$\tilde{H}_\theta = \frac{\tilde{E}_\theta}{\eta_0} \quad \propto \frac{1}{R} .$$

$$\tilde{E}_R \propto \frac{1}{R^2} \sim 0 .$$

- Power density

$$\vec{S}_{av} = \frac{1}{2} \mathrm{Re}\left(\tilde{\vec{E}} \times \tilde{\vec{H}}^* \right) = \hat{R}\, S(R, \theta, \phi)$$

$$S(R, \theta, \phi) = \frac{1}{2} \mathrm{Re}\left(\tilde{E}_\theta \tilde{H}_\phi^* \right)$$

$$= \frac{I_0^2 \ell^2 k^2 \eta_0}{32\pi^2 R^2} \sin^2\theta$$

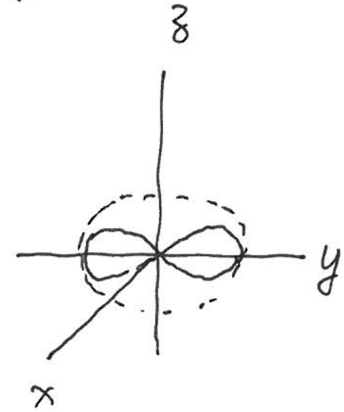
$$= S_0(R) \cdot \sin^2\theta$$

- Directional Pattern $F(\theta, \phi)$.

$$F(\theta, \phi) = \frac{S(R, \theta, \phi)}{S_{max}}$$ (Normalised radiation intensity)

$$S_{max} = \frac{I_0^2 \ell^2 k^2 \eta_0}{32\pi^2 R^2} = \frac{15\pi I_0^2}{R^2} \left(\frac{\ell}{\lambda}\right)^2$$

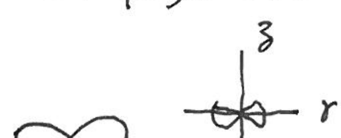
- For the Hertzian dipole.



$$F(\theta, \phi) = \sin^2\theta$$

See figure 10-8 on page 355.

constant ϕ:



constant θ. $(\theta = 90°)$: