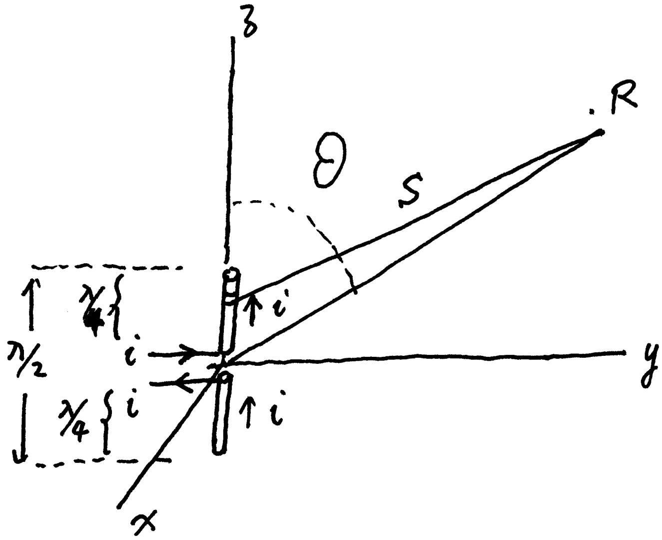
Phys 221.

Lecture 33.

- Half-wave dipole antenna



Since the current at the ends is zero.

$$i = I_0 \cos(\omega t)\cos(kz)$$
$$= \mathrm{Re}\left[I_0 \cos(kz) e^{j\omega t}\right]$$

i.e: $\tilde{I}(z) = I_0 \cos(kz)$.

(The current in the antenna is like a standing wave).

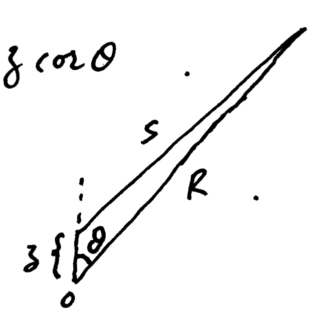
$$\tilde{I}\left(\tfrac{\lambda}{4}\right) = \tilde{I}\left(-\tfrac{\lambda}{4}\right) = 0.$$

- Analysis: Use the Hertzian dipole as an elemental current source.

Hertzian dipole : $l I_0$

Now : $\tilde{I}(z)\, dz$. — Then integrate over dz.

- Far field results: $R \gg \lambda$, $s \simeq R - z\cos\theta$.



$$d\tilde{E}_\theta = \frac{jk\eta_0}{4\pi}\tilde{I}(z)\,dz\,\frac{e^{-jks}}{s}\sin\theta$$

$$d\tilde{H}_\phi = \frac{d\tilde{E}_\theta}{\eta_0}$$

- Note: $R \gg \lambda$, $S \gg \lambda$.

but $S - R \approx z\cos\theta$ can be comparable to λ (or greater than λ)

$$\therefore \quad \frac{e^{-jkS}}{S} \approx \frac{e^{-jkR}}{R} \cdot e^{jkz\cos\theta}$$

- $$\tilde{E}_\theta = \int_{-\frac{\lambda}{4}}^{\lambda/4} \frac{jk\eta_0}{4\pi} I_0 \cdot \cos(kz) \frac{e^{-jkR}}{R} \sin\theta \cdot e^{jkz\cos\theta} \, dz.$$

$$= j\,60\, I_0 \frac{e^{-jkR}}{R} \sin\theta \cdot \int_{-\lambda/4}^{\lambda/4} \cos(kz)\, e^{jkz\cos\theta} \, dz.$$

$$= j\,60 I_0 \frac{e^{-jkR}}{R} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right]$$

$$\left[\int e^{ax}\cos(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\sin bx - b\cos bx\right) \right]$$

$$\tilde{H}_\phi = \frac{\tilde{E}_\theta}{\eta_0}$$

$\eta_0 = 120\pi$.

Both $\tilde{E}_\theta$ and $\tilde{H}_\phi$ are ind. of ϕ.

- Time-average power density.

$$S(R,\theta) = \frac{1}{2}\,\mathrm{Re}\left[\tilde{E}_\theta \tilde{H}_\phi^*\right] = \frac{1}{2\eta_0}\tilde{E}_\theta \tilde{E}_\theta^* = \frac{1}{2\eta_0}|\tilde{E}_\theta|^2$$

$$= \frac{(60 I_0)^2}{2\eta_0}\frac{1}{R^2}\frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}$$

$$= \frac{15 I_0^2}{\pi R^2}\frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}$$

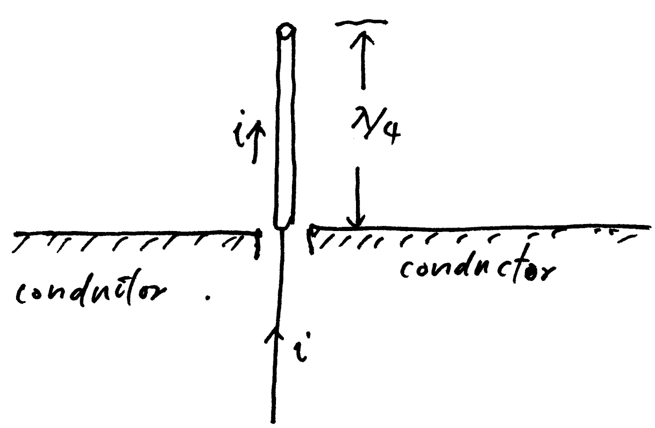
$$S_{max} = S_0 = \frac{15 I_0^2}{\pi R^2}.$$

$$F(\theta,\phi) = F(\theta) = \frac{S(R,\theta)}{S_0} = \left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2 \qquad (10.55)$$

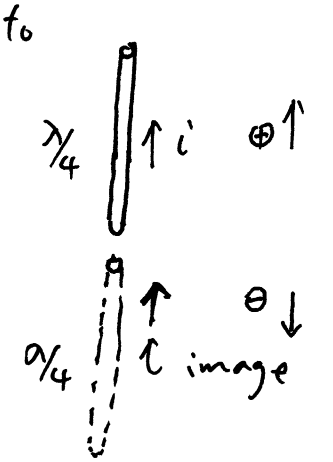
(Normalized radiation intensity).

- Quarter-wave Monopole antenna.

Idea: use the image method.



It's equivalent to



That's a $\frac{\lambda}{2}$ dipole antenna!

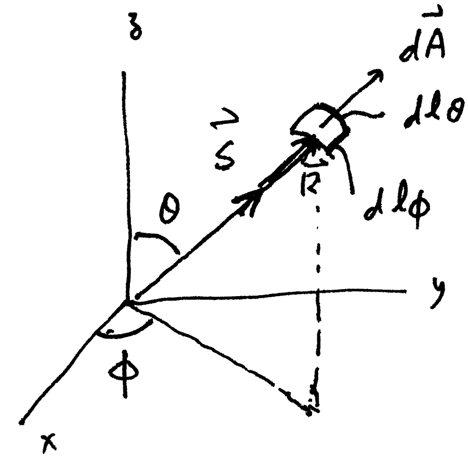
but only in the upper-half plane: $0 \le \theta \le \frac{\pi}{2}$.

- Differential power radiated by antenna through an elemental area dA.

$$dl_\theta = R\,d\theta$$

$$dl_\phi = R\sin\theta \cdot d\phi$$

$$dA = R^2 \sin\theta \cdot d\theta \cdot d\phi$$



$$dP_{rad.} = \vec{S}_{av} \cdot d\vec{A}$$

$$= (\hat{R} S) \cdot (\hat{R}\, dA) = S\, dA$$

- Solid angle associated with dA.

$$d\Omega = \frac{dA}{R^2} = \sin\theta \cdot d\theta \cdot d\phi. \qquad (dA = R^2 d\Omega)$$

(in sr. — steradians)

$$dP_{rad} = \underbrace{R^2 S(R, \theta, \phi)}\, d\Omega.$$

power per unit solid angle.

- Total radiated power.

$$P_{rad} = R^2 \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} S(R, \theta, \phi)\, d\Omega = R^2 S_{max} \iint F(\theta, \phi)\, d\Omega$$

$$= R^2 S_{max} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} F(\theta, \phi) \sin\theta\, d\theta\, d\phi.$$

Note: Total solid angle of a ~~sp~~ sphere : $= 4\pi$.

$$\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} d\Omega = 4\pi .$$

OR: $$\frac{\text{surface area of sphere}}{R^2} = \frac{4\pi R^2}{R^2} = 4\pi .$$

- Antenna pattern
  - described by the normalised radiation intensity.

$$F(\theta,\phi) = \frac{S(R,\theta,\phi)}{S_{max}} .$$

How to plot $F(\theta,\phi)$ — Next.