

Lecture 34 .

Antenna Radiation Characteristics

- Antenna pattern .

$$F(\theta, \phi) = \frac{S(R, \theta, \phi)}{S_{\max}} .$$

e.g. Hertzian dipole : $F(\theta, \phi) = \sin^2 \theta$.

- Beamwidth (at half-power, or 3dB)

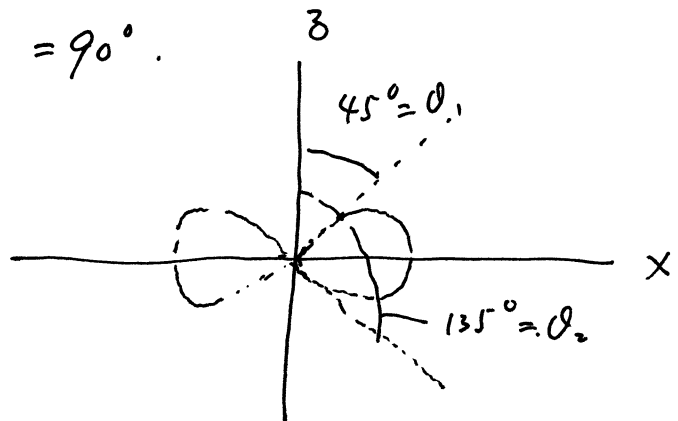
$$\beta = \Delta \theta = \theta_2 - \theta_1 . \quad (\theta_1, \theta_2 \text{ — 3dB points})$$

e.g. Hertzian dipole $F(\theta, \phi) = \sin^2 \theta = F(\theta)$.

3dB . (half - power) : $\sin^2 \theta = \frac{1}{2}$.

$$\theta_1 = 45^\circ , \quad \theta_2 = 135^\circ$$

$$\therefore \beta = 135^\circ - 45^\circ = 90^\circ .$$



• Directivity

$$D = \frac{F_{\max}}{F_{\text{av}}} = \frac{1}{\frac{1}{4\pi} \iint_{4\pi} F(\theta, \phi) d\Omega} = \frac{4\pi}{\Omega_p}$$

$$\Omega_p = \iint_{4\pi} F(\theta, \phi) d\Omega \quad \text{--- pattern solid angle}$$

e.g.: For an isotropic antenna,

$$F(\theta, \phi) = 1, \quad \Omega_p = 4\pi, \quad D = 1.$$

e.g. Hertzian dipole: $F(\theta, \phi) = \sin^2 \theta$

$$\text{Pattern solid angle: } \Omega_p = \iint \sin^2 \theta \cdot \sin \theta \cdot d\theta \cdot d\phi$$

$$= \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi} \sin^3 \theta d\theta \cdot d\phi$$

$$= \frac{8\pi}{3}$$

$$D = \frac{4\pi}{\frac{8\pi}{3}} = \frac{3}{2} = 1.5$$

~~e.g. $\lambda/2$ antenna: $F(\theta, \phi) = F(\theta) = \cos^2\left(\frac{\pi}{2} \cos \theta\right)$~~

• Radiation efficiency .

$$\xi = \frac{P_{\text{rad}}}{P_t} = \frac{\text{Power radiated into space}}{\text{Transmitter power .}}$$

$$P_{\text{rad}} = R^2 \iint S(R, \theta, \phi) \sin \theta d\theta d\phi \quad \left(= \oint S(R, \theta, \phi) dA \right)$$

$$= R^2 S_{\text{max}} \iint_{4\pi} F(\theta, \phi) d\Omega = R^2 S_{\text{max}} \Omega_p$$

$$= \frac{4\pi R^2 \cdot S_{\text{max}}}{D}$$

• Antenna gain .

$$G = \frac{4\pi R^2 S_{\text{max}}}{P_t} \quad \left(= \frac{4\pi R^2 S_{\text{max}}}{P_{\text{rad}}/\xi} = \frac{4\pi R^2 S_{\text{max}} \xi}{R^2 S_{\text{max}} \Omega_p} \right)$$

$$G = \xi D \quad \left(= \frac{4\pi \xi}{\Omega_p} \right)$$


$$= \xi D \quad \left(D = \frac{4\pi}{\Omega_p} \right)$$

• Radiation Resistance

$$P_{\text{rad}} = \frac{1}{2} R_{\text{rad}} \cdot I_0^2 = P_{\text{rad}} .$$

I_0 — amplitude .
(NOT RMS)

$$P_t = P_{\text{rad}} + P_{\text{loss}}$$

$\uparrow$
 Transmitter power
 supplied by generator

power
 radiated
 into space .

power
 dissipated
 as heat in antenna .

• Loss resistance R_{loss} :

$$\frac{1}{2} I_0^2 R_{loss} = P_{loss}$$

$$\begin{aligned} \zeta &= \frac{P_{rad}}{P_t} = \frac{P_{rad}}{P_{rad} + P_{loss}} = \frac{\frac{1}{2} I_0^2 R_{rad}}{\frac{1}{2} I_0^2 R_{rad} + \frac{1}{2} I_0^2 R_{loss}} \\ &= \frac{R_{rad}}{R_{rad} + R_{loss}} \end{aligned}$$

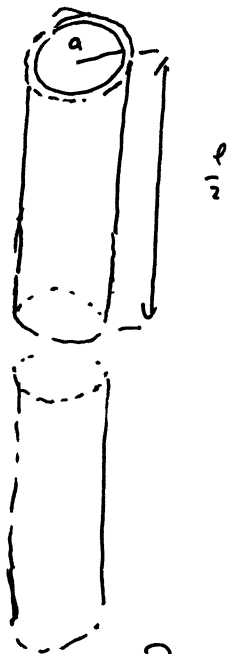
e.g. A Hertzian dipole: $l = 0.04 \text{ m}$, $f = 75 \text{ MHz} = 7.5 \times 10^7 \text{ Hz}$ $\left\{ \frac{l}{\lambda} < \frac{1}{50} \right.$
 $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{7.5 \times 10^7} = 4 \text{ m}$
 Copper: $\sigma_c = 5.8 \times 10^7 \text{ S/m}$, radius: $a = 4 \times 10^{-4} \text{ m} = 0.4 \text{ mm}$

find ζ :

$$R_{loss} = \frac{l}{2\pi a \delta_s \sigma_c}, \quad \delta_s = \sqrt{\pi f \mu_c \sigma_c}$$

$$= 0.036 \Omega$$

$$\begin{aligned} P_{rad} &= \frac{4\pi R^2}{D} S_{max} \\ &= \frac{4\pi R^2}{1.5} \cdot \frac{15\pi I_0^2}{R^2} \cdot \left(\frac{l}{\lambda}\right)^2 \\ &= 40\pi^2 I_0^2 \left(\frac{l}{\lambda}\right)^2 \end{aligned}$$



compare to: $P_{rad} = \frac{1}{2} I_0^2 R_{rad}$, $R_{rad} = \frac{2 P_{rad}}{I_0^2}$

$\therefore$ Radiation resistance: $R_{rad} = 80\pi^2 \left(\frac{l^2}{\lambda^2}\right) = 0.08 \Omega$

Radiation efficiency: $\eta = \frac{P_{\text{rad}}}{P_t} = \frac{P_{\text{rad}}}{P_{\text{rad}} + P_{\text{loss}}}$

$$= \frac{R_{\text{rad}}}{R_{\text{rad}} + R_{\text{loss}}}$$

$$= 0.69.$$

e.g. If we change the length to $l = 2\text{m}$.

it becomes a $\frac{\lambda}{2}$ dipole. ($l = \frac{\lambda}{2}$).

$R_{\text{loss}} = \text{1.8 } \Omega$. (50 times longer than it was before)

Directivity: $D = \frac{4\pi R^2 S_{\text{max}}}{P_{\text{rad}}}$ $\left(S_{\text{max}} = \frac{15 I_0^2}{\pi R^2} \right)$

$$P_{\text{rad}} = R^2 \iint S(R, \theta) d\Omega = \frac{15 I_0^2}{\pi} \int_0^{2\pi} d\phi \int_0^{\pi} \underbrace{\frac{\cos^2(\frac{\pi}{2} \cos \theta)}{\sin^2 \theta}}_{\text{can't be integrated.}} d\theta$$

$$= 36.6 I_0^2.$$

Has to be done numerically.

$$D = \frac{4\pi R^2 \cdot 15 I_0^2}{36.6 I_0^2 \cdot \pi R^2} = 1.64.$$

$$(10 \log 1.64 = 2.15 \text{ dB})$$

Radiation resistance: $R_{\text{rad}} = \frac{P_{\text{rad}}}{\frac{1}{2} I_0^2} = \frac{36.6 I_0^2}{0.5 I_0^2} = 73 \Omega.$

Radiation efficiency:

$$\eta = \frac{R_{\text{rad}}}{R_{\text{rad}} + R_{\text{loss}}} = \frac{73}{73 + 1.8} \approx 98\%.$$

Better than the Hertzian dipole (69%).

More important, it's much easier to match the impedance:

$$\frac{\lambda}{2} \text{ dipole: } R_{\text{rad}} + R_{\text{loss}} \approx 75 \Omega.$$

$$\text{Hertzian: } R_{\text{rad}} + R_{\text{loss}} \approx 0.1 \Omega \cdot \left(\begin{array}{l} \text{too small,} \\ \text{Hard to} \\ \text{match} \end{array} \right)$$