

Assignment 1. Solutions.

1. Page 44. #2.5.

Given: $\vec{A} = \hat{x} + \hat{y}2 - \hat{z}3$, $\vec{B} = \hat{x}3 - \hat{y}4$, $\vec{C} = \hat{y}3 - \hat{z}4$.

$$(a) \quad A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{14}$$

$$\hat{A} = \frac{\vec{A}}{A} = \frac{1}{\sqrt{14}} (\hat{x} + \hat{y}2 - \hat{z}3)$$

$$(b) \quad \vec{B} \cdot \hat{C} = \vec{B} \cdot \frac{\vec{C}}{C} = \frac{B_x C_x + B_y C_y + B_z C_z}{\sqrt{C_x^2 + C_y^2 + C_z^2}} = \frac{(-4)(3)}{\sqrt{3^2 + (-4)^2}} = -\frac{12}{5}$$

$$(c) \quad \vec{A} \cdot \vec{C} = AC \cos \theta_{AC}.$$

$$\cos \theta_{AC} = \frac{\vec{A} \cdot \vec{C}}{AC} = \frac{(2)(3) + (-3)(-4)}{\sqrt{14} \cdot 5} = 0.9621$$

$$\theta_{AC} = \cos^{-1} 0.9621 = 15.8^\circ$$

$$(d) \quad \vec{A} \times \vec{C} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & -3 \\ 0 & 3 & -4 \end{vmatrix} = \hat{x}(-8+9) - \hat{y}(-4) + \hat{z}(3)$$

$$= \hat{x} + \hat{y}4 + \hat{z}3.$$

$$(e) \quad \vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \begin{vmatrix} 1 & 2 & -3 \\ 3 & -4 & 0 \\ 0 & 3 & -4 \end{vmatrix}$$

$$= 16 - 2(-4)(3) + \cancel{(-3)}(3)(3)$$

$$= 16 + 24 - 27$$

$$= 13$$

$$(f) \quad \vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

$$= (\hat{x}3 - \hat{y}4) [2 \times 3 + (-3)(-4)] - (\hat{y}3 - \hat{z}4)(3-8)$$

$$= (\hat{x}3 - \hat{y}4)(18) + (\hat{y}3 - \hat{z}4)5$$

$$= \hat{x}54 - \hat{y}57 - \hat{z}20$$

$$(g) \quad \hat{x} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 0 & 0 \\ 3 & -4 & 0 \end{vmatrix} = \hat{z}(1)(-4) = -\hat{z}4$$

$$(h) \quad (\vec{A} \times \hat{y}) \cdot \hat{z} = \hat{z} \cdot (\vec{A} \times \hat{y}) = \begin{vmatrix} 0 & 0 & 1 \\ 1 & -2 & -3 \\ 0 & 1 & 0 \end{vmatrix} = 1$$

(a). Gaussian surface:

spherical surface $r = \frac{2}{3}R$.

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{Q}{\epsilon_0}$$

by symmetry: $\vec{E} = E \hat{R}$.On Gaussian surface: $E = \text{const}$.

$$\therefore \oint \vec{E} \cdot d\vec{s} = \oint E ds$$

$$= E \int ds = E \cdot 4\pi r^2 \quad \left(\begin{array}{l} \text{surface area of} \\ \text{a sphere: } 4\pi r^2 \end{array} \right)$$

$$\therefore E = \frac{Q}{4\pi \epsilon_0 r^2}$$

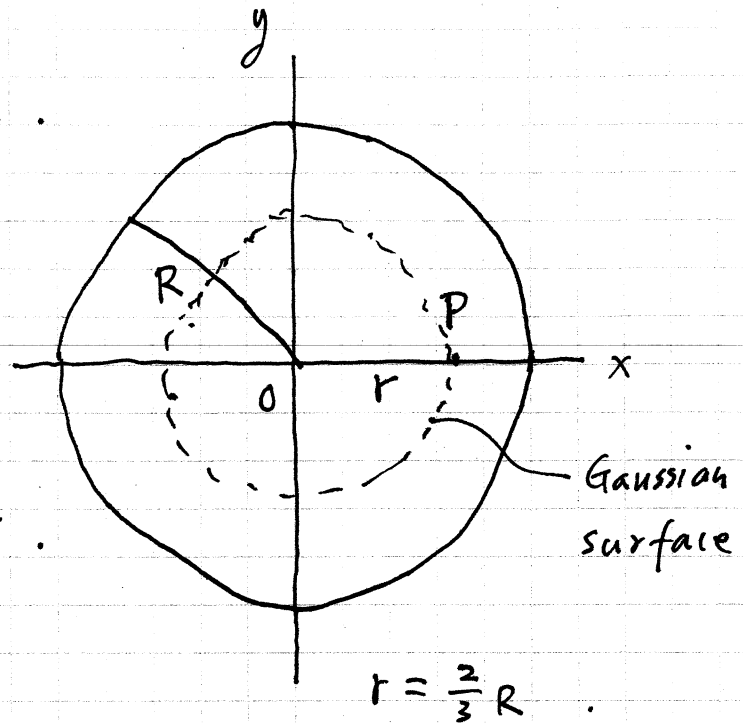
 Q — charge inside the Gaussian surface.

$$\therefore Q = \rho \cdot V = \rho \cdot \frac{4}{3}\pi r^3$$

$$\therefore E = \frac{\rho r}{3\epsilon_0} = \frac{2\rho R}{9\epsilon_0}$$

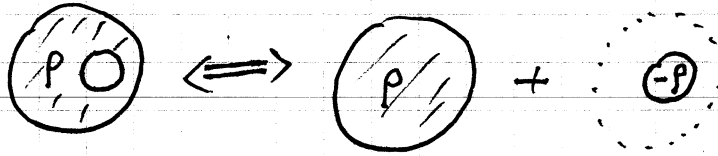
$$\text{at point P: } \vec{E} = \hat{x} \frac{2\rho R}{9\epsilon_0}$$

$$\text{at any point inside sphere: } \vec{E} = \frac{\rho \vec{r}}{3\epsilon_0}$$



(b). Creating a cavity is equivalent to adding a negatively charged object.

i.e.:



$$\therefore \vec{E} = E_p + E_{-p}$$

at point P: $\vec{E} = E_x \hat{x}$

$$E_x = E_x^+ + E_x^-$$

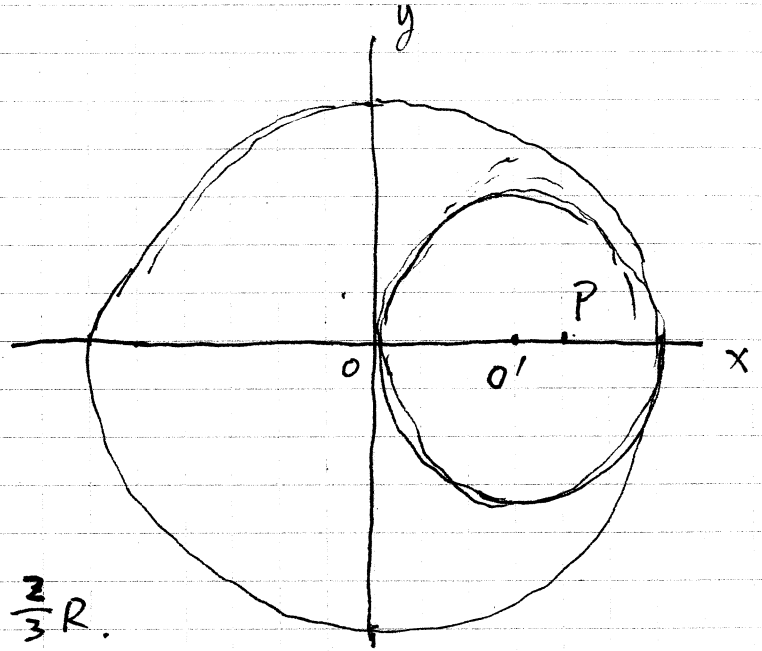
$$= \frac{\rho r^+}{3\epsilon_0} - \frac{\rho r^-}{3\epsilon_0}$$

r^+ — from O to P. $r^+ = \frac{2}{3}R$.

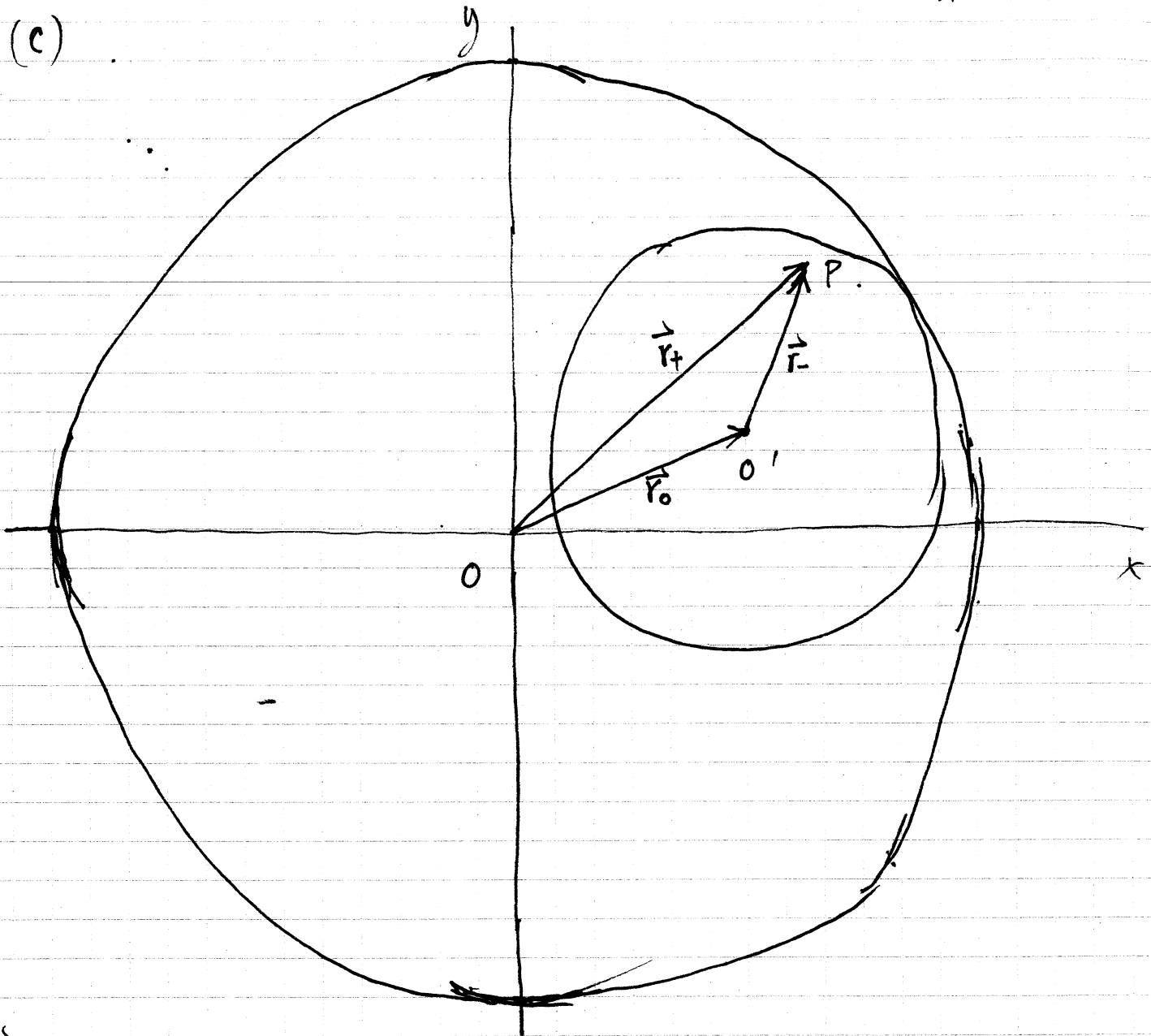
r^- — from O' to P: $r^- = (\frac{2}{3}R - \frac{1}{2}R) = \frac{1}{6}R$.

$$\therefore E_x = \frac{\rho}{3\epsilon_0} \left(\frac{2}{3}R - \frac{1}{6}R \right) = \frac{\rho R}{6\epsilon_0}$$

at P: $\therefore \vec{E} = \hat{x} \frac{\rho R}{6\epsilon_0}$



(c)



$\vec{E}$ field at P:

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{\rho}{3\epsilon_0} \vec{r}_+ - \frac{\rho}{3\epsilon_0} \vec{r}_- = \frac{\rho}{3\epsilon_0} (\vec{r}_+ - \vec{r}_-)$$

but: $\vec{r}_+ - \vec{r}_- = \vec{r}_0 = \text{position vector of centre of cavity.}$

$$\therefore \vec{E} = \frac{\rho \vec{r}_0}{3\epsilon_0}$$