

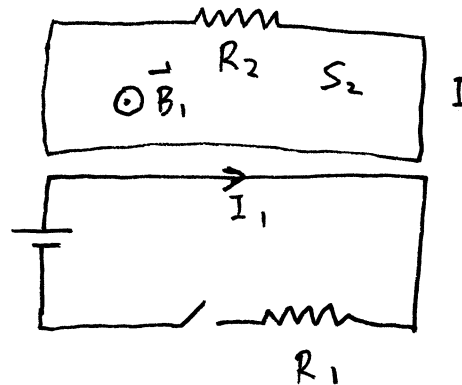
Assignment #8 solutions.

6.1

at $t = 0$

$$\Phi_{12} = \int_{S_2} \vec{B}_1 \cdot d\vec{s}$$

is increasing.

The induced current I should reduce the total $\vec{B}$ -field in loop 2.i.e. against $\vec{B}_1$.Therefore I should be clockwise.at $t = t_1$, I_1 is reduced to zero. Φ_{12} is decreasing.The induced current I should increase the $\vec{B}$ -field in 2.i.e., same direction as $\vec{B}_1$.Therefore, I should be counterclockwise.

$\vec{B}_1$ — Magnetic field due to I_1 .

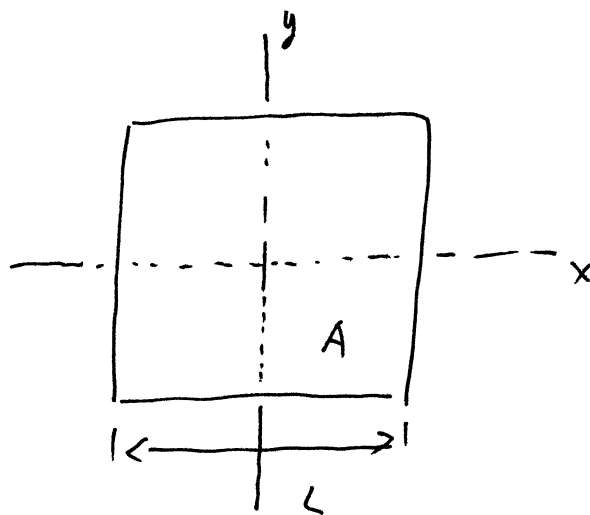
6.3. $V_{emf} = -N \frac{d\Phi}{dt}$ $N=100$

a). $\vec{B} = \hat{z} 10 e^{-2t}$ $\Phi = BA$

$$V_{emf} = -NA \frac{dB}{dt}$$

$$= - (100)(L^2) \cdot 10(-2) e^{-2t}$$

$$= 125 e^{-2t} \text{ (V)}$$



$$L = 0.25 \text{ m}$$

$$A = L^2 = 0.0625 \text{ m}^2$$

b). $\vec{B} = \hat{z} 10 \cdot \cos x \cdot \cos(10^3 t)$

$$\Phi = \int \vec{B} \cdot d\vec{s} = 10 \cdot \cos(10^3 t) \cdot 2 \cdot \int_0^{0.125} \cos x \cdot dx \cdot (0.25)$$

$$= -5 \cdot \cos(10^3 t) \cdot \sin(0.125)$$

$$V_{emf} = -N \cdot \frac{d\Phi}{dt} = -100 \cdot (-5) \cdot \sin(0.125) \cdot (-\sin 10^3 t) \cdot 10^3$$

$$= 62.5 \cdot \sin(10^3 t) \text{ (KV)}$$

c). $\vec{B} = \hat{z} 10 \cdot \cos x \cdot \sin 2y \cdot \cos 10^3 t$

Since $\sin(2y)$ is an odd function

$$\Phi = \int \vec{B} \cdot d\vec{s} = 0 \quad (\text{ind. of time})$$

$$\therefore V_{emf} = 0$$

6.5

$$V_{emf} = - \frac{d\Phi}{dt} = - A \frac{dB}{dt}$$

$$B = B_0 \cos(2\pi f t)$$

$$\frac{dB}{dt} = -2\pi f B_0 \sin(2\pi f t)$$

$$V_{emf} = 2\pi f A B_0 \sin(2\pi f t)$$

$$V_{emf}(\max) = 2\pi f A B_0$$

$$B_0 = \frac{20 \text{ mV}}{2\pi \cdot 3 \times 10^8 \times 0.01}$$

$$B_0 = 1 \times 10^{-9} \text{ T}$$

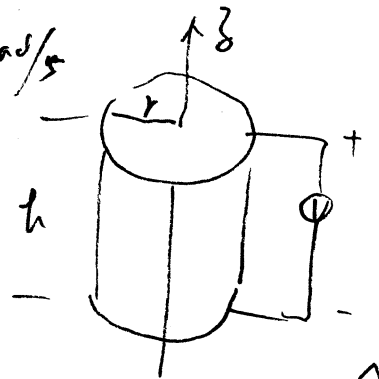
6.11

$$1200 \text{ RPM} = \omega = \frac{2\pi \cdot 1200}{60} \text{ rad/s}$$

$$\vec{B} = \hat{y} 6 \text{ (T)}$$

$$r = 0.05 \text{ m}$$

$$h = 0.1 \text{ m}$$



$$u = r\omega = (0.05) 40\pi \text{ m/s} = 2\pi \text{ m/s} \quad \vec{u} = 2\pi \hat{\phi}$$

$$\begin{aligned} V_{emf} &= \int (\vec{u} \times \vec{B}) \cdot d\vec{\ell} = -uB \cdot h = (2\pi)(6)(\cancel{0.05} 0.1) \\ &= -1.2\pi \text{ (V)} \\ &= -3.77 \text{ V} \end{aligned}$$

6.18 .

$$\nabla \cdot \vec{J} = - \frac{\partial \rho_v}{\partial t} .$$

$$\nabla \cdot \vec{J} = \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} .$$

$$= 0 - 6y \cos \omega t .$$

$$\text{i.e. } \frac{\partial \rho_v}{\partial t} = 6y \cos \omega t .$$

$$\rho_v = \int 6y \cos \omega t \, dt$$

$$= \frac{6y}{\omega} \sin \omega t + C .$$