PHYS 101 Final Examination (Version B)

December 16, 2002 Name Key

Time: 3 hours Student No.______________________

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Part I (#1-15): For each of the following fifteen questions, please circle one answer only.

a

1(3%). Two athletes jump straight up. John has twice the initial speed of Harry. Compared to Harry, John jumps

a. four times as high
b. twice as high
c. three times as high
d. 1.41 times as high
e. 70% as high

$mgh = \frac{1}{2}mv^2$

$h \propto v^2$

c

2(3%). An object is moving with a constant velocity. Which of the following statements must be true:

a. A constant force is being applied in the direction of motion
b. There are no forces acting on the object
c. The net force acting on the object is zero
d. There is no frictional force acting on the object
e. The net force acting on the object depends on the magnitude of the velocity

$\vec{F}_{net} = 0$

e

3(3%). A person applies a constant force of 20 N to a rock of 1000 kg for 20 seconds. What is the work done by this person if the rock does not move at all by this applied force?

a. 1000 J
b. 20,000 J
c. 400,000 J
d. 400 J
e. 0

$\vec{F} \cdot \Delta\vec{x} = 0$ if $\Delta\vec{x} = 0$

d

4(3%). When a parachutist jumps from an airplane, he eventually reaches a constant speed, called the terminal velocity. This means that

a. the acceleration is equal to g
b. the force of air resistance is equal to zero
c. the effect of gravity has died down
d. the force of air resistance is equal to the weight of the parachutist
e. the mechanical energy of the parachutist is conserved

$\uparrow \vec{F}_{drag}$ $\quad \vec{F}_{net} = 0$

$\downarrow m\vec{g}$ $\quad F_{drag} = mg$

b

5(3%). A golf club exerts an average force of 1000 N on a 0.045-kg golf ball which is initially at rest. The club is in contact with the ball for 1.8 ms. What is the speed of the golf ball as it leaves the tee?

a. 35 m/s
b. 40 m/s
c. 45 m/s
d. 50 m/s
e. 12 m/s

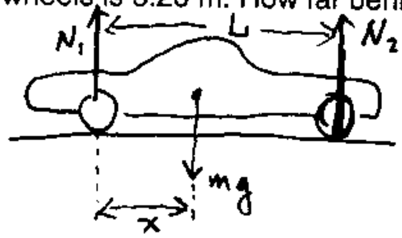
$F\Delta t = \Delta p = mv$

$v = \frac{F \cdot \Delta t}{m} = \frac{1000 \times 0.0018}{0.045} = 40 \text{ m/s}$

c

6(3%). To determine the location of the center of mass of a car, the car is driven over a scale. When the front wheels are over the scale, the weight recorded by the scale is 5800 N, and when the rear wheels are over the scale, the scale reads 6500 N. The distance between the front and rear wheels is 3.20 m. How far behind the front wheels is the center of mass located?

a. 1.50 m
b. 1.59 m
c. 1.69 m
d. 1.72 m
e. 1.60 m



$N_1 + N_2 = mg$.

$mg\,x = N_2 L$.

$x = \frac{N_2 L}{mg} = \frac{N_2 L}{N_1 + N_2} = \frac{6500 \times 3.2}{6500+5800} = 1.69$

e

7(3%). Two spherical balls are made of the same material. Ball A has a radius R, while ball B has 2R. If ball A has a moment of inertia I, what is the moment of inertia of ball B?

a. 2I
b. 4I
c. 8I
d. 16I
e. 32I

$I = \frac{2}{5} M R^2 = \frac{2}{5} \rho \cdot V R^2 = \frac{2}{5} \rho \cdot \frac{4}{3} \pi R^3 \cdot R^2 \propto R^5$.

$2^5 = 32$

e

8(3%). An object attached to a spring oscillates with an amplitude of 5cm and a period of 0.5s. What is the maximum speed of the object?

a. 10.0 m/s
b. 0.31 m/s
c. 0.10 m/s
d. 0.025 m/s
e. 0.63 m/s

$v_{max} = \omega A = \frac{2\pi}{T} A = \frac{(2\pi)(0.05)}{0.5} = (2\pi)(0.1) \approx 0.63$

a

9(3%). Four waves are described by the following expressions, where distances are measured in meters and times in seconds

I $y = 0.12 \cos(3x - 21t)$ $= 0.12 \cos[3(x - 7t)]$ $v = +7$ m/s
II $y = 0.15 \sin(6x + 42t)$ $= 0.15 \sin[6(x + 7t)]$ $v = -7$ m/s.
III $y = 0.13 \cos(6x + 21t)$ $= 0.13 \cos[6(x + 3.5t)]$ $v = -3.5$ m/s
IV $y = -0.23 \sin(3x - 42t)$ $= -0.23 \sin[3(x - 14t)]$ $v = 14$ m/s

Which of these waves have the same speed?

a. I and II
b. II and III
c. III and IV
d. II and IV
e. They all have different speeds.

b

10(3%). As you stand by the side of the road, a car approaches you at a constant speed, sounding its horn, and you hear a frequency of 76 Hz. After the car goes by, you hear a frequency of 65 Hz. What is the speed of the car? The speed of sound in air is 343 m/s.

a. 26 m/s
b. 27 m/s
c. 28 m/s
d. 29 m/s
e. 30 m/s

$76 = f / (1 - \frac{u}{v})$ $65 = f / (1 + u/v)$.

$76(1 - \frac{u}{v}) = 65(1 + \frac{u}{v})$

$11 = 141 \frac{u}{v}$. $u = \frac{11 v}{141} = \frac{(11)(343)}{141} = 26.76 \approx 27$ m/s e

11(3%). One of the harmonics of a string fixed at both ends has a frequency of 52.2 Hz and the next higher harmonic has a frequency of 60.9 Hz. What is the fundamental frequency of the string?

a. 26.1 Hz
b. 17.4 Hz
c. 30.4 Hz
d. 113 Hz
e. 8.7 Hz

$L = \frac{\lambda}{2} \cdot n$. $\quad \lambda_n = \frac{2L}{n}$. $\quad f_n = \frac{v}{\lambda_n} = n\frac{v}{2L}$.

$f = \frac{1}{T} = \frac{v}{\lambda} \quad f_1 = \frac{v}{2L} = f_{n+1} - f_n$.

$\therefore f_1 = 60.9 - 52.2 = 8.7\ Hz$

e.

12(3%). When you hold your hand palm-up, the atmosphere exerts a force of about 800 N on the palm of your hand. Comparing this force with

I. the force on the back of the hand
II. the force on the palm when you are holding your hand vertical
III. the force on the palm when it is palm-down

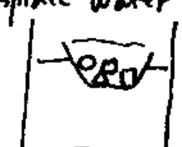
a. It is different from I, II, and III
b. It is equal to I but different from II and III
c. It is equal to II and III but different from I
d. It is equal to III but different from I and II
e. They are all equal

a.

13(3%). A boat loaded with rocks is floating in a swimming pool. If the rocks are thrown into the pool, the water level in the pool, after the rocks have settle to the bottom,

a. falls
b. rises
c. stays the same
d. rises if the rocks are large, falls if the rocks are small.
e. There is not enough information to answer this question.

displace water



in the same weight.

water displaced

have the same volume

d

14(3%). The filament in a light bulb has a diameter of 0.050 mm and an emissivity e = 1.00. The temperature of the filament is 3000°C. How long should the filament be in order to radiate 60 W of power? The Stefan-Boltzmann constant is σ=5.67 x 10^{-8}.

a. 2.9 cm
b. 8.6 cm
c. 7.2 cm
d. 5.9 cm
e. 2.4 cm

$P = e \cdot \sigma A T^4$.

$= e\sigma \pi D \cdot l \cdot T^4$.

$\therefore l = \frac{P}{e\sigma\pi D T^4}$

$= \frac{60}{\pi\, 5.67\times10^{-8} \times 0.05\times10^{-3} \times (3273.15)^4}$

$= 0.0587\ m$

c

15(3%). Motor oil, with a viscosity of 0.250 Ns/m^2, is flowing through a tube that has a radius of 3.00 mm and is 1.00 m long. The drop in pressure is 200 kPa. What is the average speed of the oil?

a. 0.70m/s
b. 0.80 m/s
c. 0.90 m/s
d. 1.00 m/s
e. 1.10 m/s

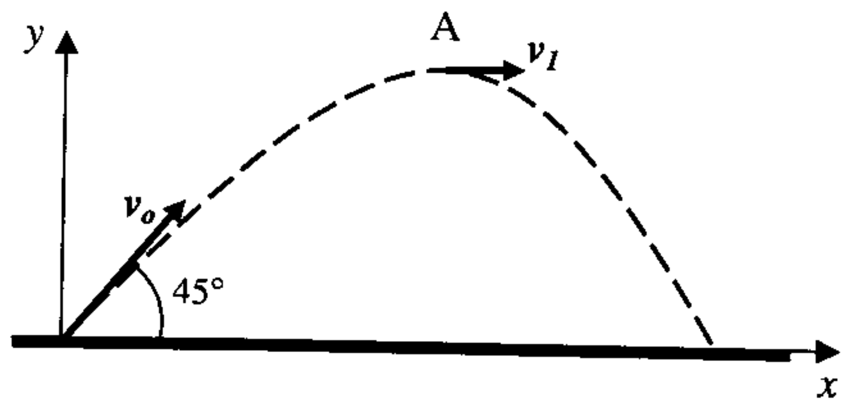
$\Delta P = 8\pi\eta \frac{lV}{A}$

$V = \frac{A \cdot \Delta P}{8\pi\eta l} = \frac{\cancel{\pi}(0.003)^2 \cdot 200000}{8\cancel{\pi}(0.25)(1)} = 0.9\ m/s$

Part II (#16-22): For questions 16 to 22, please show complete solutions and explain your reasoning, stating any principles that you have used.

16(8%) . A ball of mass 0.5 kg is kicked into the air at an angle of 45° with an initial velocity of 30m/s. 2.0 seconds later, it reaches its maximum height of 12m at point A with a velocity of 10.0m/s to the right.

a) Find the average acceleration of the ball in the first 2.0 seconds.
b) Find the work done by the drag due to air resistance in the first 2.0 seconds.

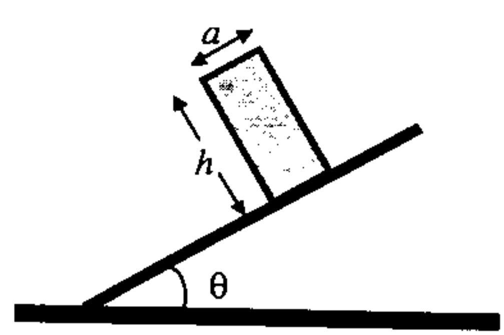


17(7%). A cart of mass m moving to the right with a velocity v collides elastically with a cart of mass $3m$ moving to the left with a velocity of $-v$. What are the final velocities of the two carts?

18(8%). A tall, uniform, rectangular block sits on an inclined plane as shown in the figure below. The coefficient of static friction is μ_s=0.4 and the width of the block is a=20cm.

(a) If $h=3a$, does the block slide or fall over as the angle θ is slowly increased?

(b) To prevent the block from falling over, the height of the block must be kept below a certain value. Find the maximum height h such that the block will not fall over before it slides.



19(8%). A pencil, 16 cm long, is released from a vertical position with the eraser end resting on a table. The eraser does not slip. Treat the pencil like a uniform rod.

(a) What is the angular acceleration of the pencil when it makes a 30° angle with the vertical?

(b) What is the angular speed of the pencil when it makes a 30° angle with the vertical?

20(8%). A bird watcher listens to the sound of a singing bird. When he is 10 m from the bird, he hears the sound with an intensity level of 50 dB. The radius of his eardrum is about 4.0 mm.

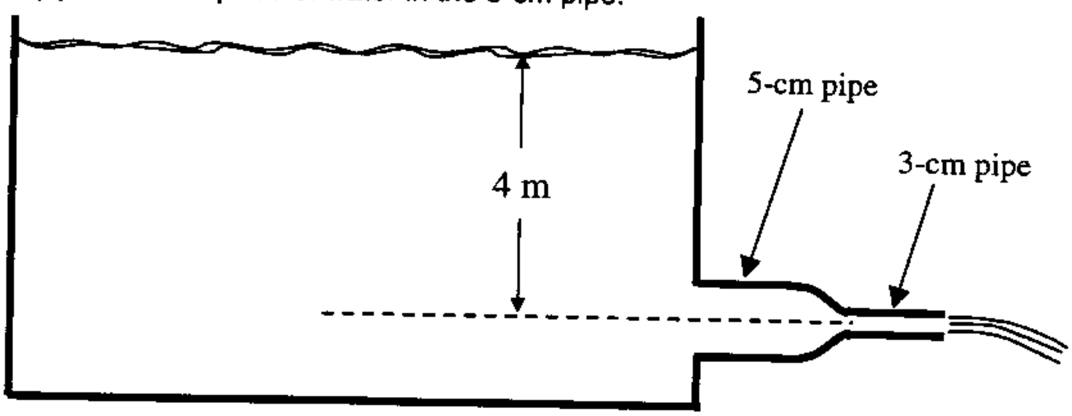
(a) Find the power of sound received by the eardrum of the bird watcher.

(b) Find the intensity level of the sound at a position that is 100 m from the bird.

21(8%). Water flows out of a large reservoir through a 5-cm diameter pipe. The pipe is connected to a 3-cm diameter pipe that is open to the atmosphere, as shown in the figure below. The pipes are 4m below the water level of the reservoir.

(a) Find the speed of water in the 3-cm pipe.

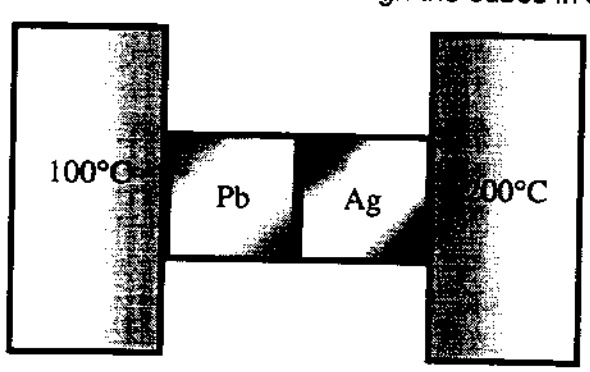
(b) Find the speed of water in the 5-cm pipe.



22(8%). The figure below shows two metal cubes, with 2-cm sides, between two walls, one held at 100°C and the other at 200°C. The cubes are lead and silver, whose thermal conductivities are 353 W/m·K and 429 W/m·K respectively.

(a) Find the temperature at the lead-silver junction.

(b) Find the amount of heat that flows through the cubes in 5.0 s.



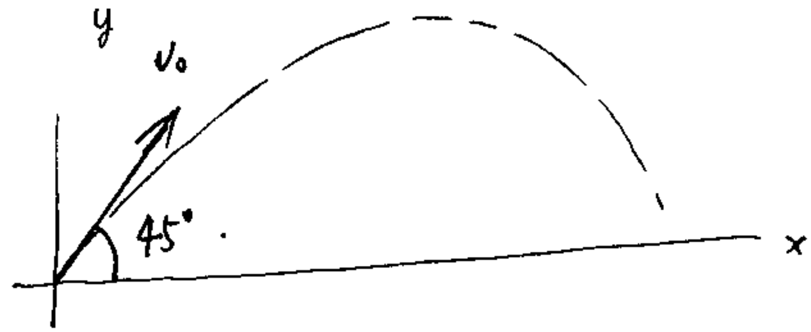
#16 .

$\vec{v}_i$: $v_{0x} = V_0 \cos 45°$

$= 21.2$ m/s .

$V_{0y} = V_0 \sin 45°$

$= 21.2$ m/s .



$\vec{v}_f$: $v_x = 10$ m/s .

$V_y = 0$.

a): average acc: $\vec{a} = \dfrac{\Delta \vec{V}}{\Delta t}$

$\Delta V_x = 10 - 21.2 = -11.2$ m/s .

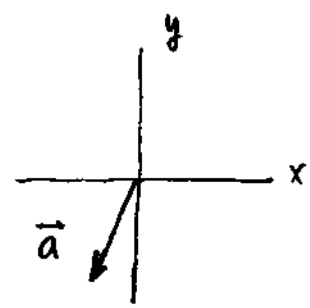
$\Delta V_y = 0 - 21.2 = -21.2$ m/s .

$a_x = \dfrac{\Delta V_x}{\Delta t} = \dfrac{-11.2}{2.0} = -5.6$ m/s²

$a_y = \dfrac{\Delta V_y}{\Delta t} = \dfrac{-21.2}{2.0} = -10.6$ m/s²

$a = \sqrt{(-5.6)^2 + (-10.6)^2} = 12$ m/s² .

$\theta = \tan^{-1} \dfrac{a_y}{a_x} = 62.1° + 180° = 242°$.



b). $W = \Delta U + \Delta K$ (work-energy Thm) .

$\Delta U = mgh = (0.5)(9.81)(12) = 58.8$ J .

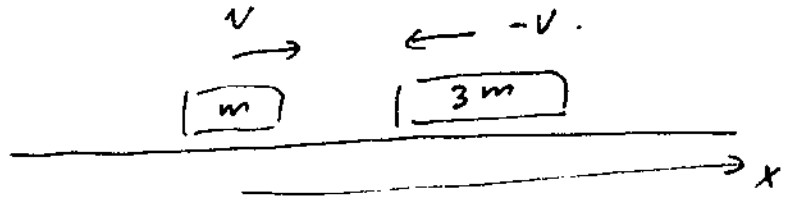
$\Delta K = \frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2 = \frac{1}{2} m (V_f^2 - V_i^2)$

$= \frac{1}{2} (0.5)(10^2 - 30^2) = -200$ J

$W = 58.8 - 200 = -141.2$ J .

17.

elastic collision:



Total Momentum is conserved:

$$\vec{P}_f = \vec{P}_i \quad : \qquad mv - 3mv = mv_1' + 3mv_2' \qquad (1)$$

(where v_1' = velocity of m after collision
v_2' = velocity of $3m$ after collission)

Total Kinetic Energy is conserved:

$$K_i = K_f : \qquad \frac{1}{2}mv^2 + \frac{1}{2}(3m)v^2 = \frac{1}{2}m v_1'^2 + \frac{1}{2}(3m)v_2'^2 \qquad (2).$$

Simplify (1) and (2):

$$-2v = v_1' + 3v_2' \qquad (3)$$

$$4v^2 = v_1'^2 + 3v_2'^2 \qquad (4).$$

Square (3):

$$4v^2 = v_1'^2 + 6v_1'v_2' + 9v_2'^2 \qquad (5)$$

(5) − (4):

$$0 = 6v_1'v_2' + 6v_2'^2.$$

i.e: $v_2'(v_1' + v_2') = 0$.

solution 1: $\begin{cases} v_2' = 0, \\ v_1' = -2v. \end{cases}$ (because of (3)).

solution 2: $\begin{cases} v_1' + v_2' = 0. \quad \text{i.e. } -2v = 2v_2' \quad \text{(because of (3))} \\ \text{OR: } v_2' = -v, \quad v_1' = v. \end{cases}$

Solution 2 should be rejected because it means no collision.

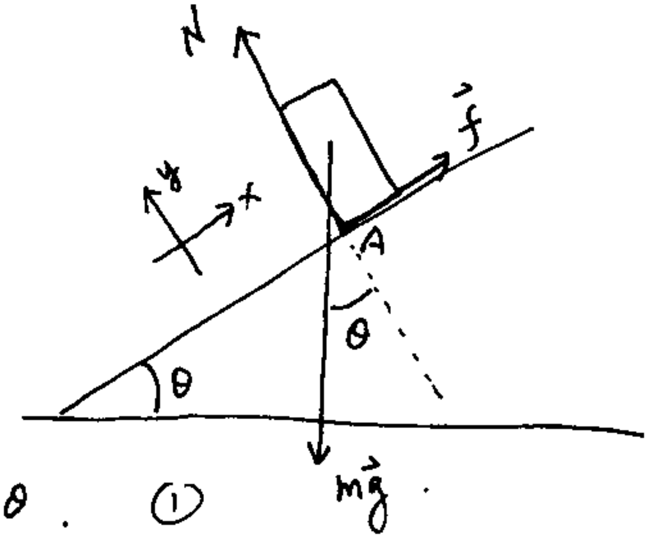
∴ Solution 1 is the answer.

i.e: m will go back with a velocity $-2v$.
and $3m$ will stop.

18.

$N = mg\cdot\cos\theta$.

When the block were about to slide,



$$mg\sin\theta = \mu\cdot mg\cdot\cos\theta \quad ①$$

The net torque is: (about point A).

$$\tau = mg\sin\theta\cdot\frac{h}{2} - mg\cos\theta\cdot\frac{a}{2}$$

$$= \mu\cdot mg\cdot\cos\theta\cdot\frac{h}{2} - mg\cdot\cos\theta\cdot\frac{a}{2}$$

$$\tau = \frac{1}{2}mg\cdot\cos\theta\cdot(\mu h - a) \quad ②$$

(a). now, $\mu = 0.4$, $h = 3a$.

$\therefore\ \tau = \frac{1}{2}mg\cdot\cos\theta\,[(0.4)\cdot 3a - a] > 0$.

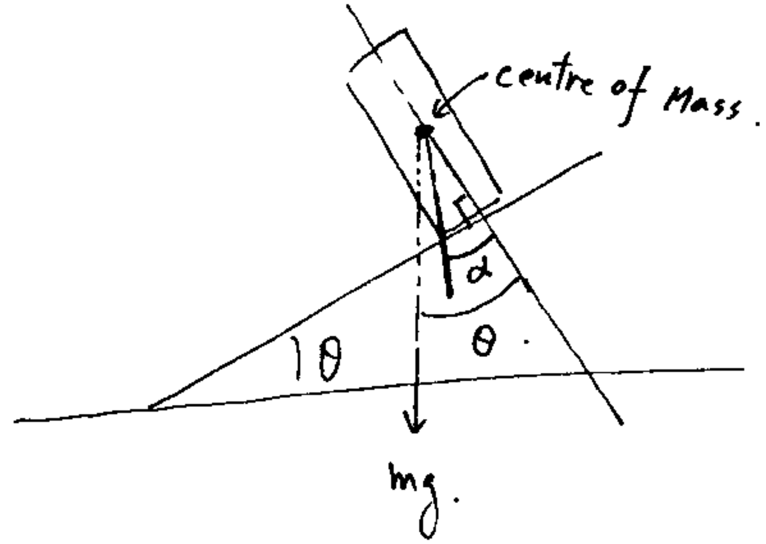
$\therefore$ the block falls over before it slides.

(b). The maximum hight is given by $\tau = 0$.

i.e. $\mu h - a = 0$. (see eq. ②)

$$h = \frac{a}{\mu} = \frac{20\text{ cm}}{0.4} = 50\text{ cm}.$$

18. (Another approach).



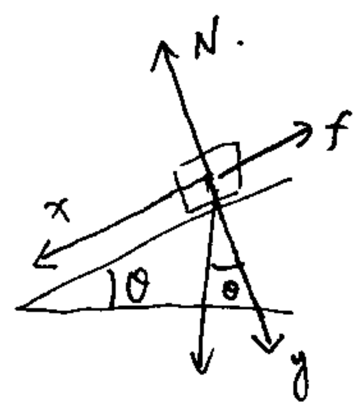
As shown in the figure.

$$\alpha = \tan^{-1}\frac{a/2}{h/2} = \tan^{-1}\frac{a}{h}.$$

when $\theta > \alpha$, the gravitational force, which goes through the centre of mass of the block, will fall outside the base of the block and result in a torque causing the block to tip over.

Therefore, when $\left\{\begin{array}{l}\theta > \alpha \\ \tan\theta > \alpha\end{array}\right\}$, the block will fall over.

Now, we need to find out when the block starts to slide. the x-component of gravity is:



$$W_x = mg \cdot \sin\theta.$$

The normal force

$$N = mg\cos\theta.$$

The maximum static frictional force

$$f = N\mu_s = \mu_s \cdot mg \cdot \cos\theta.$$

When. $W_x > f$, the block will slide.

i.e: $mg\sin\theta > \mu_s mg\cos\theta$.

OR: $\tan\theta > \mu_s$. i.e. $\theta > \tan^{-1}\mu_s$.

a). Now, $\tan\alpha = \dfrac{a/2}{h/2} = \dfrac{a}{h} = \dfrac{a}{3a} = \dfrac{1}{3} = 0.33$.

$\mu_s = 0.4$.

When θ is increased slowly, $\tan\theta$ will reach $\tan\alpha$ before reaching μ_s. Therefore, the block will fall over.

b). To prevent the block from falling over,

We need $\tan\alpha > \mu_s$.

i.e: $\dfrac{a}{h} > 0.4$.

$$h < \frac{a}{0.4} = \frac{10}{4}a = \frac{5}{2}a.$$

$\therefore$ The maximum height is $h_{max} = \frac{5}{2}a$.

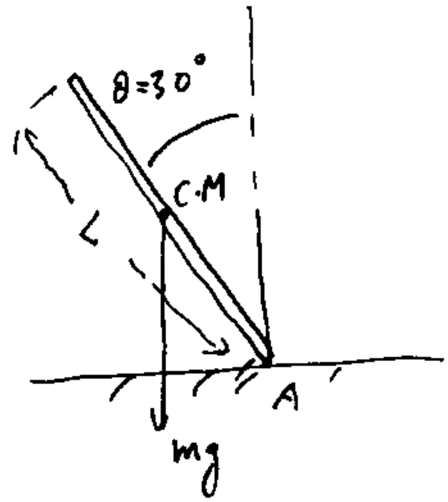
$$= \frac{5}{2}(20\text{ cm})$$
$$= 50\text{ cm}.$$

19. Torque about point A:

$\tau = mg \cdot \frac{L}{2} \sin\theta = \frac{1}{4} mgL$.

~~Moment~~
moment of Inertia about A:

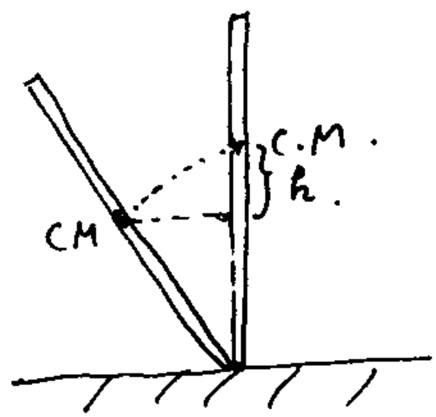
$I = \frac{1}{3} m L^2$.



(a): $\tau = I \cdot \alpha$. i.e: $\alpha = \frac{\tau}{I}$.

$$\alpha = \frac{1}{4}mgL \Big/ \frac{1}{3}mL^2 = \frac{3}{4}\frac{g}{L} = \frac{3 \times 9.81}{4 \times 0.16} = 46 \text{ rad/s}^2.$$

(b). The height of the centre of mass is dropped by $h = \frac{L}{2} \cdot (1 - \cos\theta)$



Mechanical energy conservation:

$$mgh = \frac{1}{2} I \cdot \omega^2 = \frac{1}{2} \left(\frac{1}{3}\right) \cdot m L^2 \cdot \omega^2.$$

i.e: $g \frac{L}{2} (1 - \cos\theta) = \frac{1}{6} L^2 \omega^2$.

$$\omega^2 = \frac{3g(1-\cos\theta)}{L}$$

$$\omega = \sqrt{\frac{3g(1-\cos\theta)}{L}} = \sqrt{\frac{3(9.81)(1-\cos 30^\circ)}{0.16}} = 4.96 \text{ rad/s}$$

20.

(a). $\beta = 10 \log_{10}\left(\frac{I}{I_0}\right) \qquad I_0 = 10^{-12}\ W/m^2$

$$50 = 10 \log_{10}\left(\frac{I}{I_0}\right)$$

$$5 = \log_{10}\left(\frac{I}{I_0}\right)$$

$$10^5 = \frac{I}{I_0}.$$

$$I = 10^5 \cdot I_0 = 10^5 \times 10^{-12} = 10^{-7}\ W/m^2.$$

$$P = I \cdot A = I \cdot \pi\left(\frac{d}{2}\right)^2 = 10^{-7} \times \pi\left(\frac{0.004}{2}\right)^2 = 1.26 \times 10^{-12}\ W.$$

(b). Assuming the source power is P_0.

$\boxed{\cancel{I_{100} = \frac{P_0}{4\pi R^2}}}$

The sound intensity at distance r is:

$$I_1 = \frac{P_0}{4\pi r^2} \qquad (r = 10\ m).$$

when the distance is $10m = 10r$:

$$I_2 = \frac{P_0}{4\pi(10r)^2} = \frac{P_0}{100(4\pi r^2)} = \frac{I_1}{100}.$$

The intensity level:

$$\beta_{100m} = 10 \log_{10}\left(\frac{I_2}{I_0}\right) = 10 \cdot \log_{10}\left(\frac{I_1}{100 I_0}\right)$$

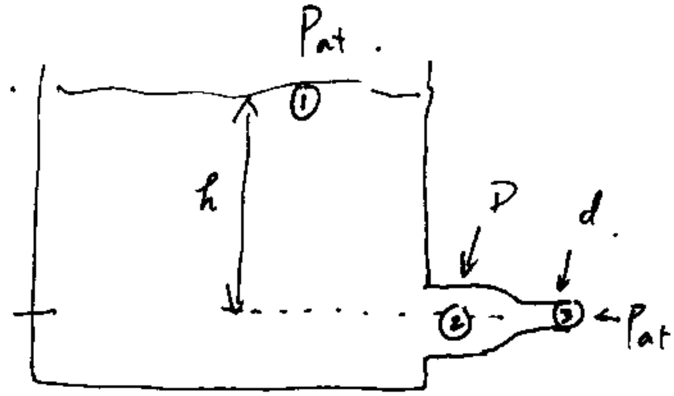
$$= 10\left[\log\left(\frac{I_1}{I_0}\right) - \log_{10}(100)\right]$$

$$= 10 \log\left(\frac{I_1}{I_0}\right) - 10 \times 2.$$

$$= 50\ dB - 20\ dB = 30\ dB.$$

21.

(a) $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_3 + \rho g h_3 + \frac{1}{2}\rho v_3^2$.



$P_1 = P_3$, $h_1 - h_3 = h$, $v_1 = 0$.

$\therefore \quad \rho g h = \frac{1}{2}\rho v_3^2$

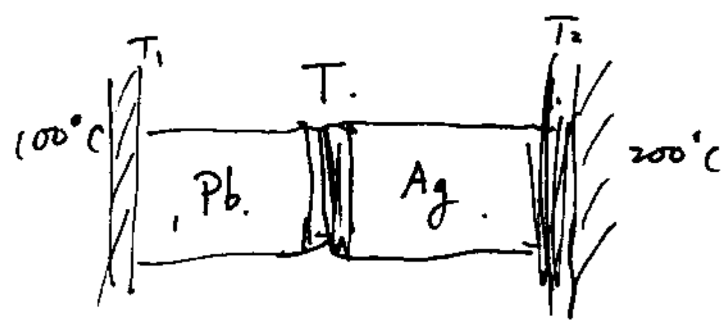
$$v_3 = \sqrt{2gh} = \sqrt{2(9.81)(4)} = 8.86 \text{ m/s}$$

(b). $A_2 \cdot v_2 = A_3 v_3$.

$$v_2 = \frac{A_3}{A_2} \cdot v_3 = \frac{3^2}{5^2} \cdot 8.86 = 3.19 \text{ m/s}.$$

22.

a) $H_1 = k_1 \frac{A \Delta T_1}{L} = H_2 = k_2 \frac{A \Delta T_2}{L}$



$k_1(T - T_1) = k_2(T_2 - T)$

$k_1 T - k_1 T_1 = k_2 T_2 - k_2 T.$

$$T = \frac{k_1 T_1 + k_2 T_2}{k_1 + k_2} = \frac{(353)(100) + 429(200)}{353 + 429} \approx 155\ ^\circ C.$$

$(154.86\,^\circ C)$

b). $H = H_1 = H_2 = (353)(0.02)(154.86^\circ - 100^\circ C)$

$= 387$ Watts (J/s)

$Q = H \cdot \Delta t = (387)(5) = 1935$ J.