Physics 101.

Lecture 2. Wed. Jan 7, 2004

Topics : 1-D Kinematics

Displacement, velocity, acceleration

Ref: WALKER. Ch.2.

- Last lecture : Instantaneous velocity. (velocity)

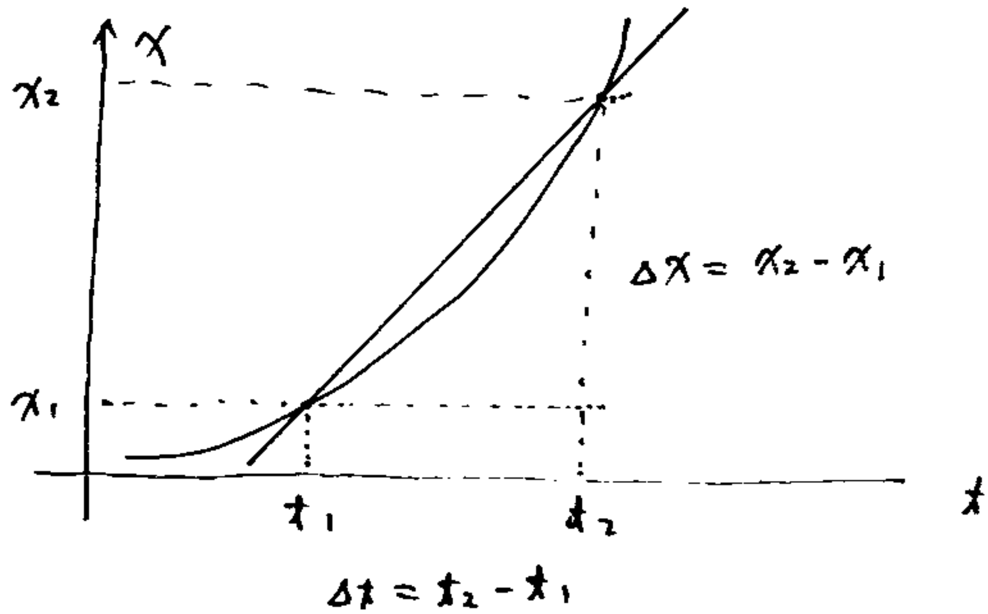
$$v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}.$$ (Time-rate of change in x)

speed : $$S \equiv \lim_{\Delta t \to 0} \frac{d}{\Delta t} = \lim_{\Delta t \to 0} \frac{|\Delta x|}{\Delta t}.$$

i.e.: $S = |v|$. ($\because$ within Δt (small) the direction of motion is not changed.)

$|\Delta x| = d$ for small Δt.

- Meaning of v and v_{av} on x-t graph.



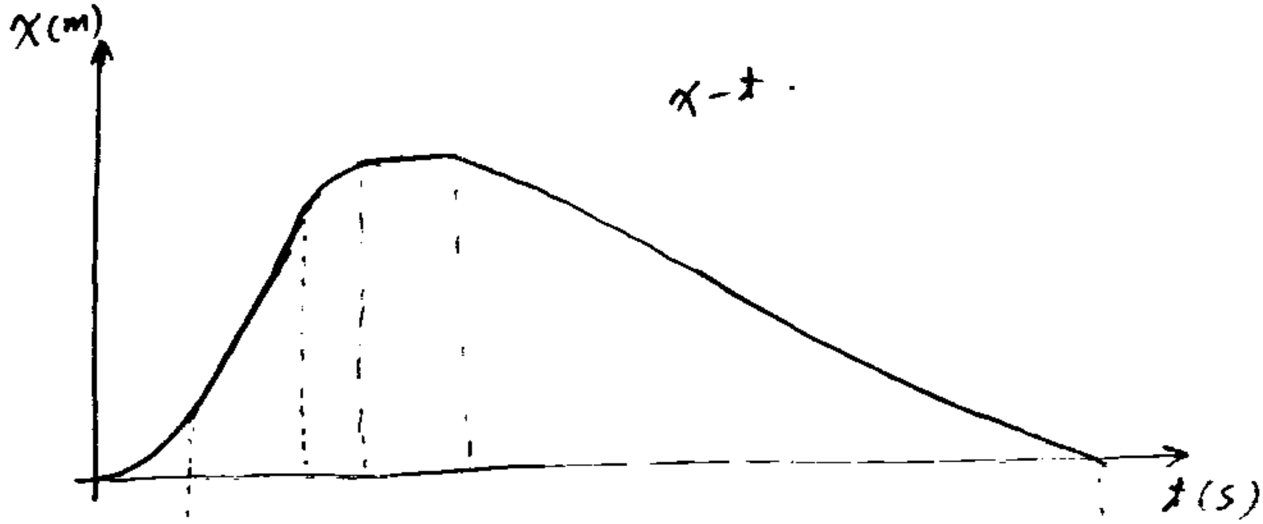
Average velocity : $v_{av} = \frac{\Delta x}{\Delta t}$

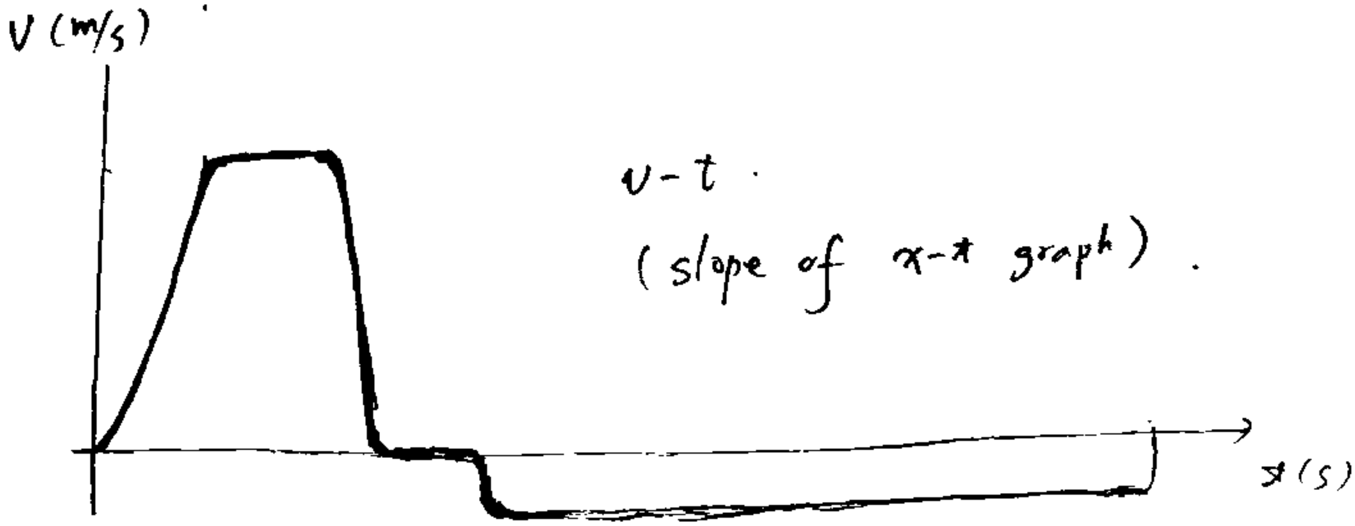
= slope of secant line

Instantaneous Velocity $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$.

= slope of tangent line.

e.g. The 100-m runner.





- How to find the displacement from a given v-t graph.

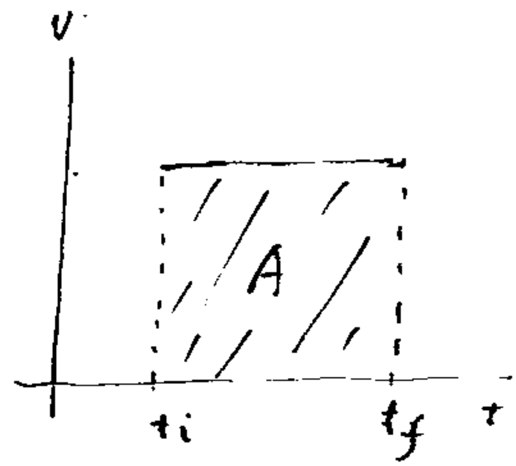
  — Δx = area under the v-t curve.

why ?

- When $v = \text{const}$.

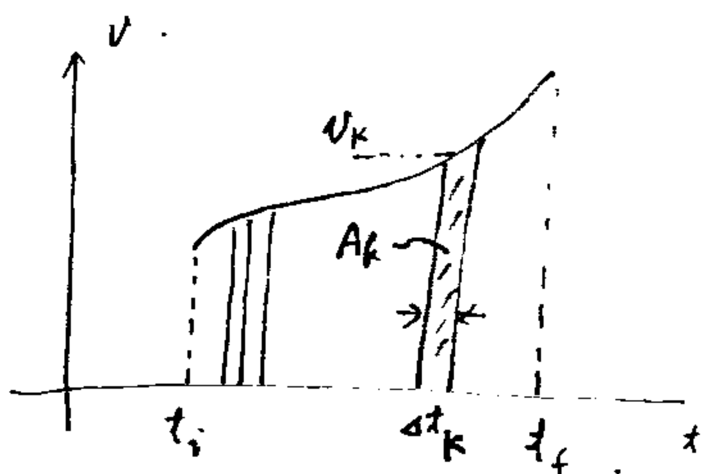
$$v = v_{av} = \frac{\Delta x}{\Delta t}.$$

$$\Delta x = v \cdot \Delta t = \text{area } A$$
$$= v\,(t_f - t_i)$$



- When $v \neq \text{const}$.

Idea: Divide the total time interval $(t_f - t_i)$ into many small intervals.



For a small time interval Δt, the velocity is approximately constant, and thus the displacement in Δt is

$$\Delta x_K \approx v_K \cdot \Delta t \approx \text{area } A_k$$

Total displacement

$$\Delta x = \Delta x_1 + \Delta x_2 + \cdots + \Delta x_k + \cdots \Delta x_N \approx \Sigma A_k.$$
$$\approx \text{Total area under the } v\text{-}t \text{ curve}.$$

when $\Delta t_k \to 0$, $\Delta x = \text{area under } v\text{-}t \text{ curve}.$

- Acceleration

In general, v varies.

To describe how quickly v is changing.

we use acceleration.

Average acceleration : $a_{av} = \frac{\Delta v}{\Delta t}$.

(Instantaneous) acceleration $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$.

time-rate change in v.

The relation between a and v is the same as the relation between v and x.

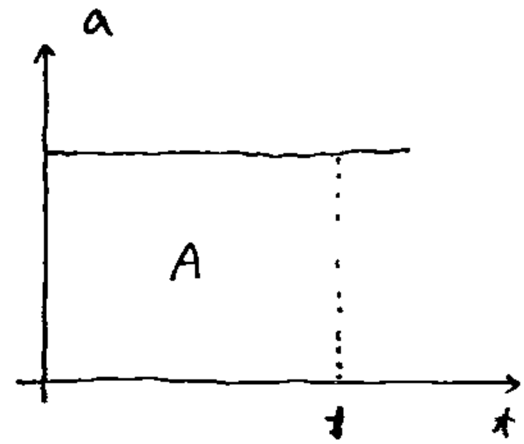
$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$	$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$
from x-t to $v(t)$ v = slope of x-t graph.	from v-t to $a(t)$. a = slope of v-t curve.
from v-t to Δx Δx = area under v-t curve.	from a-t to Δv: Δv = area under a-t curve.

e.g.: Given : $a = \text{const}$, when $t=0$, $x = x_0$, $v = v_0$.

Find : $v(t)$, $x(t)$.

Note : Here $t_i = 0$, $t_f = t$ (t – arbitrary)

$$\Delta t = t_f - t_i = t.$$



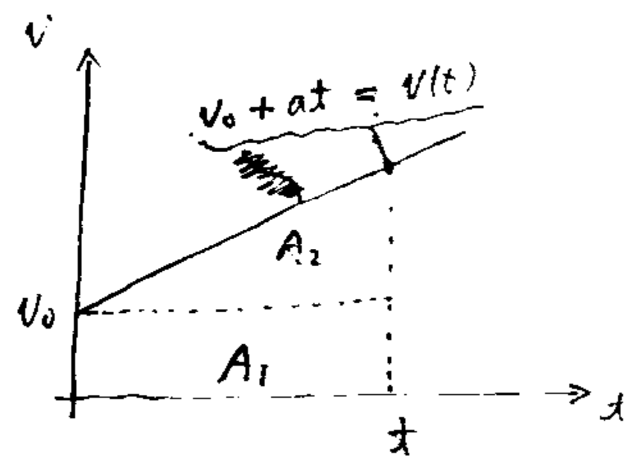
when $a = \text{const}$.

$$a = a_{av} = \frac{\Delta V}{\Delta t}$$

$$\Delta V = a \cdot \Delta t = \text{area } A.$$

$$v - v_0 = at$$

$$\boxed{v = v_0 + at}. \quad ①$$



To find $x(t)$:

Δx = area under v–t curve.

$$= \boxed{A_1} + A_2$$

$$= V_0 t + \frac{1}{2} t (v - v_0)$$

$$= V_0 t + \frac{1}{2} t [v_0 + at - v_0]$$

$$\therefore \quad \Delta x = v_0 t + \frac{1}{2} a t^2 = x - x_0$$

$$\boxed{x = x_0 + v_0 t + \frac{1}{2} a t^2}. \quad ②$$

Note : ① and ② are true only if $a = \text{const}$.