Physics 101

Lecture 4 Mon. Jan. 12, 2004

Topics : Projectile Motion
Relative Motion

Ref : Ch. 3, ch. 4.

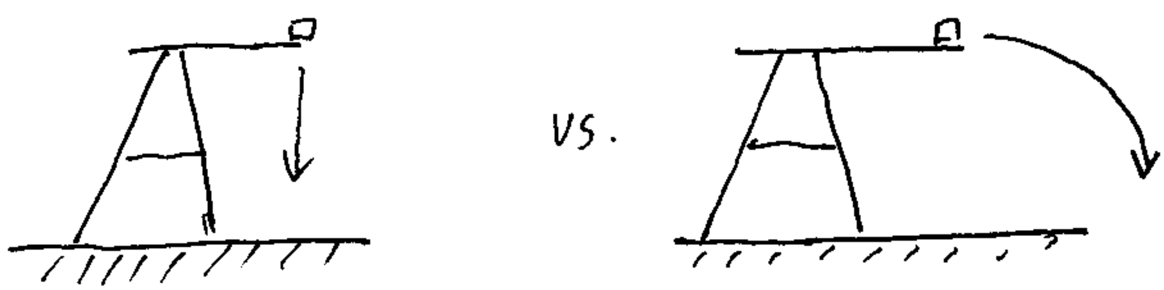
Demo : Falling blocks.

Last Lecture :

One of the advantages of using components for vectors :
x- and y- components are independent !

$$(\because \ \hat{x} \perp \hat{y})$$

Demo: Falling blocks with and without a ~~initial~~ horizontal velocity. (v_{x0}).



which one hits the ground first ?

They hit the ground at the same time!

i.e. whether or not the block moves in the horizontal direction does not affect its motion in the vertical direction.

Indeed, the vertical component of motion is independent of the horizontal component.

e.g: Projectile Motion.

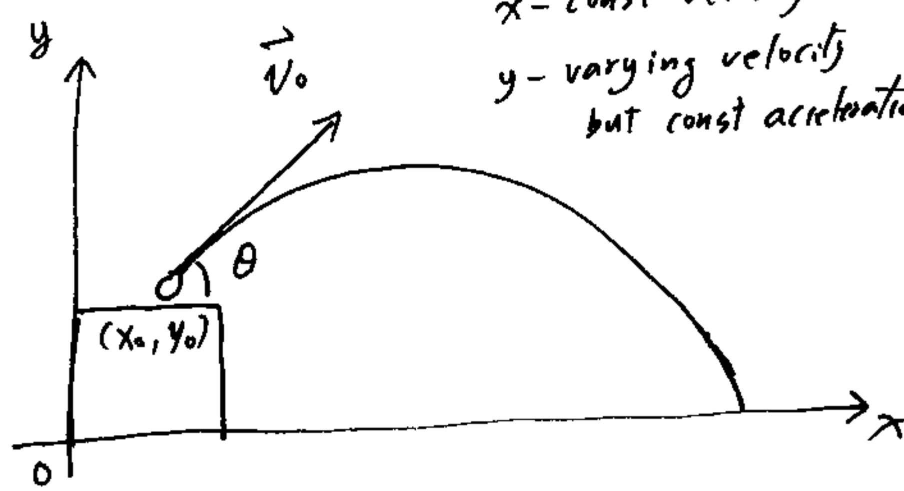
idea: Treat x- and y- components independently

acceleration $\vec{a} \parallel \hat{y}$.

x- const velocity
y- varying velocity but const acceleration

$a_x = 0$

$a_y = -g = -9.8\ m/s^2$



velocity:

$v_x = v_{0x} = v_0 \cos\theta$

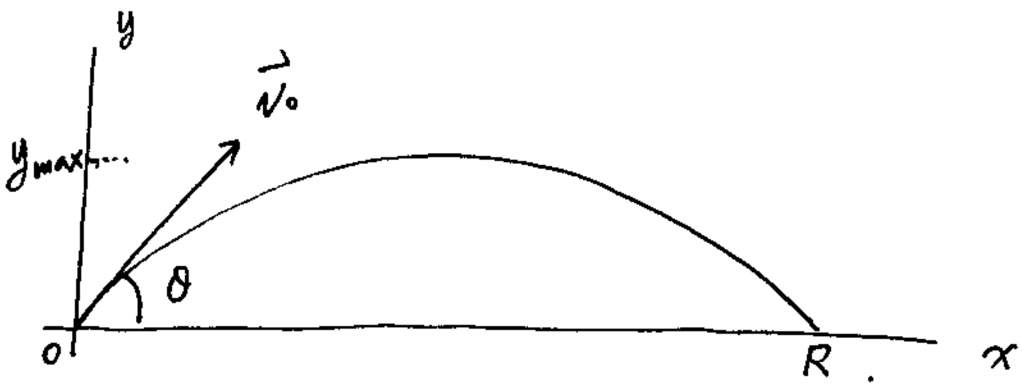
$v_y = v_{0y} + a_y t = v_0 \sin\theta - g t$

position:

$x = x_0 + v_0 \cos\theta \cdot t$

$y = y_0 + v_0 \sin\theta \cdot t - \frac{1}{2} g t^2$.

- Special case — if we can choose $x_0 = 0$
  $y_0 = 0$



$x = v_0 \cos\theta \cdot t$ | $v_x = v_0 \cos\theta$

$y = v_0 \sin\theta \cdot t - \frac{1}{2} g t^2$ | $v_y = v_0 \sin\theta - gt$.

Q1. Find the maximum height y_{max}

Key idea: when it reaches y_{max}, $v_y = 0$.

why? if $v_y > 0$, it will go higher.

if $v_y < 0$, it's not as high as before.

$\therefore$ when ~~$v_y = 0$~~ $y = y_{max}$, $v_y = 0$.

i.e: $v_0 \sin\theta - gt = 0$.

i.e: $t = v_0 \sin\theta / g$.

$$y_{max} = v_0 \sin\theta \cdot \left(v_0 \sin\theta / g \right) - \frac{1}{2} g \cdot \left(v_0 \sin\theta / g \right)^2$$

$$= \frac{1}{2} \frac{(v_0 \sin\theta)^2}{g}$$

Q2. Find the Range R.

When it hits the ground, $y=0$.

i.e., $v_0 \sin\theta \cdot t - \frac{1}{2} g t^2 = 0$.

solve for t: $t = \dfrac{2 V_0 \sin\theta}{g}$

$$x_{max} = R = v_0 \cos\theta \cdot \frac{2 V_0 \sin\theta}{g}$$

$$= \frac{V_0^2}{g} \cdot 2\sin\theta \cdot \cos\theta = \frac{V_0^2 \sin 2\theta}{g}$$

Q3: What angle θ will yield the largest Range R?

since $R = \dfrac{V_0^2}{g} \cdot \sin(2\theta)$

when $2\theta = 90°$, $\sin(2\theta) = 1 = max$

$\therefore$ $\theta = 45°$ will yield the largest range.

$$R_{max} = \frac{V_0^2}{g}. \quad (\text{if } \theta = 45°)$$

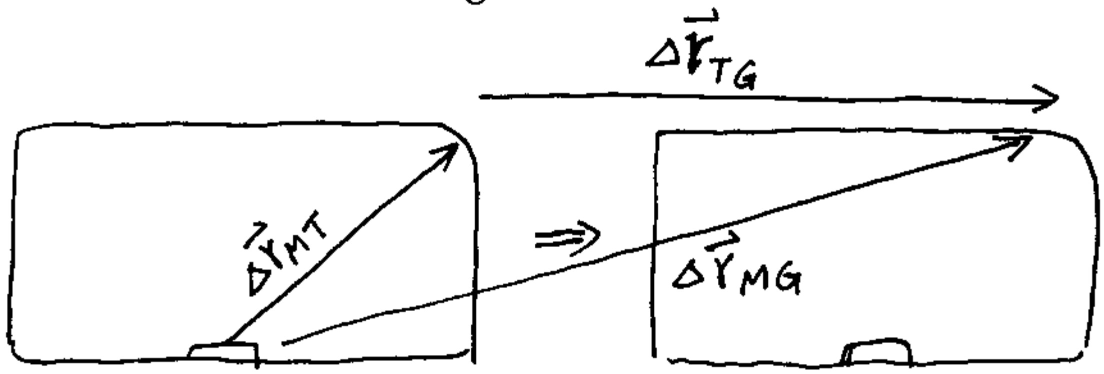
- Watch out!

Q₁, Q₂, Q₃ are just examples for $\begin{cases} x_0 = 0 \\ y_0 = 0. \end{cases}$

Sometimes $x_0 \neq 0$ OR. $y_0 \neq 0$. Then the results may be diff…

- Relative Motion

e.g. A man walking in a moving train



In time interval Δt

The man moves from door to corner : displacement $= \Delta\vec{r}_{MT}$

MT — man relative to Train

The train moves to the right by a displacement $\Delta\vec{r}_{TG}$.

TG — Train relative to Ground.

Question : How do we find the displacement of the man with repect to the ground ? $\Delta\vec{r}_{MG}$

From the diagram, and the Head-to-Tail Rule :

$$\underset{\substack{\uparrow \\ \text{absolute} \\ \text{displacement}}}{\Delta\vec{r}_{MG}} = \underset{\substack{\uparrow \\ \text{relative} \\ \text{displacement}}}{\Delta\vec{r}_{MT}} + \underset{\substack{\uparrow \\ \text{displacement} \\ \text{of the moving reference}}}{\Delta\vec{r}_{TG}} . \qquad (1) .$$

- For velocity :

$$\frac{(1)}{\Delta t} \Rightarrow \quad \vec{v}_{MG} = \vec{v}_{MT} + \vec{v}_{TG}$$

- For acceleration, similarly :

$$\vec{a}_{MG} = \vec{a}_{MT} + \vec{a}_{TG} .$$

e.g: CAPA set 1. # 9, 10, 11.