

Phys 101

Lecture 9

~~Jan. 23, 2004~~ Jan. 23, 2004. Friday

Topics Covered:

Circular Motion .

Impulse .

Work .

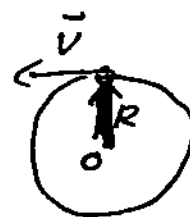
Work - Energy Theorem .

Ref: 6-5. Ch. 7

- Uniform Circular motion . $\vec{a} = \vec{a}_c$

 $\vec{a}_c$ — centripetal acceleration .magnitude $a_c = \frac{v^2}{R}$

direction : center-seeking .



- Newton's law $\vec{F}_c = m\vec{a}_c$ requires a net force $\vec{F}_c$ to provide $\vec{a}_c$

$$\vec{F}_c = m a_c = m \frac{v^2}{R}$$

called centripetal force .

physically, $\vec{F}_c$ can be

{ Tension in a string .
 gravity on a satellite .
 frictional force

between tires and Road .

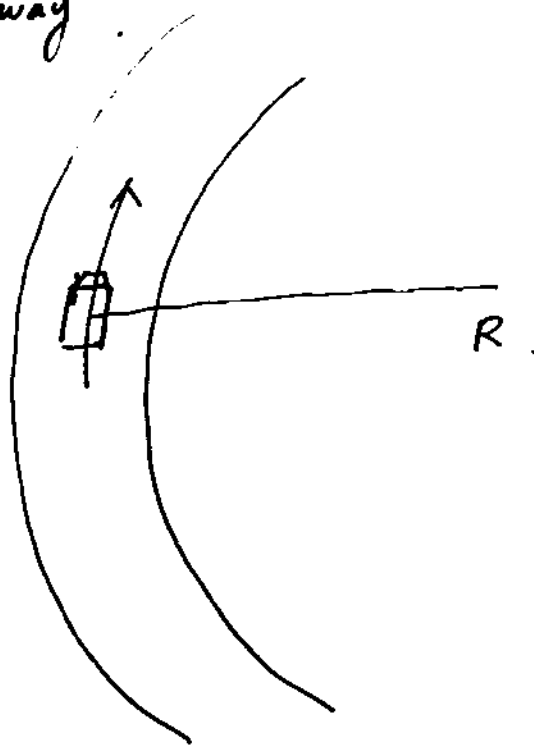
- Note: Don't count centripetal force as an additional force in the free-body diagram.

e.g: Turning on a highway.

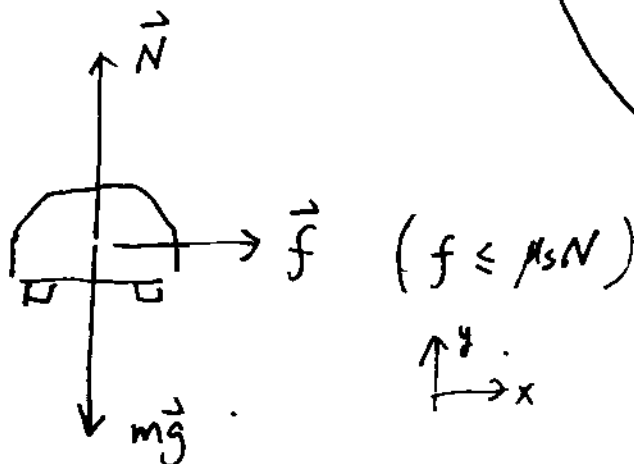
Given: $v = 80 \text{ km/h} = 22 \text{ m/s}$.

$R = 300 \text{ m}$.

Question: What is the required μ_s value?



free-body diagram:



$$\vec{F} = m\vec{a} : \quad y: \quad N = mg.$$

$$x: \quad f = m \frac{v^2}{R}.$$

$$\mu_s mg = m \frac{v^2}{R}.$$

$$\mu_s = \frac{v^2}{Rg} = \frac{22^2}{(300)(9.8)} = 0.16.$$

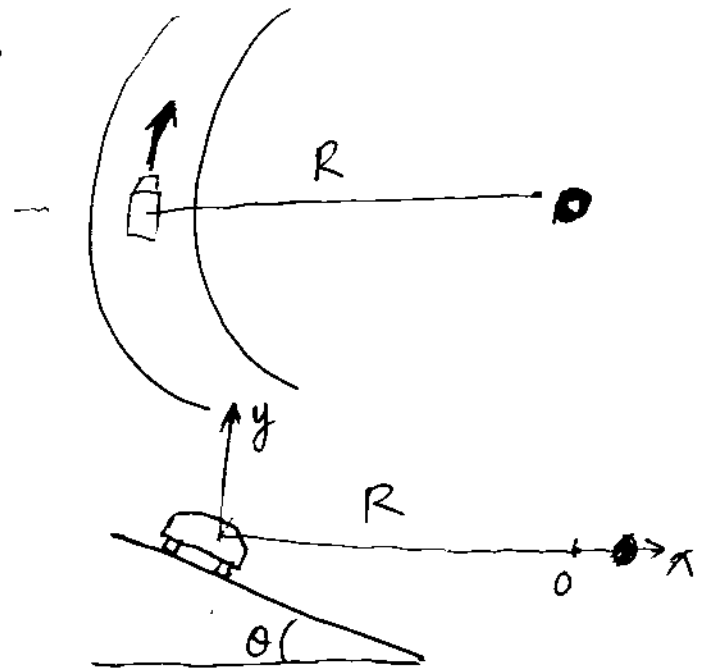
Typically
~~Normally~~, $\mu_s \sim 0.8$. good enough.

- What if μ_s is small? (e.g. icy Road surface)

How do we get the centripetal force for the turn?

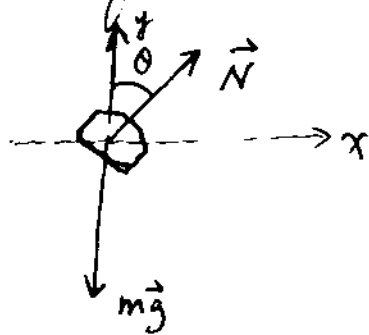
Idea: bank the highway (make the outer side higher than the inner side).

and make use of the horizontal component of the normal force.



Free-Body Diagram.

(assuming No friction)



Question: Find θ so that $N_x = F_c = ma_c = m \frac{v^2}{R}$.

y-component: $N \cos \theta - mg = 0$, $N = \frac{mg}{\cos \theta}$

x-component: $N \sin \theta = m \frac{v^2}{R}$

i.e. $\frac{mg}{\cos \theta} \cdot \sin \theta = m \frac{v^2}{R} \Rightarrow \tan \theta = \frac{v^2}{Rg}$

$\theta = \tan^{-1} \frac{v^2}{Rg} = \tan^{-1} \frac{(22)^2}{(300)(9.8)} = 9.5^\circ$

- Newton's Law : $\vec{F} = m\vec{a}$. (Instantaneous relation)

$\vec{F}$ — Net force .

~~Net~~

- Force acting on m over a small time interval Δt :

$$\begin{aligned}
 \vec{F} \cdot \Delta t &= m\vec{a} \cdot \Delta t \\
 &= m \frac{\Delta \vec{v}}{\Delta t} \cdot \Delta t \\
 &= m \Delta \vec{v} \\
 &= m (\vec{v}_f - \vec{v}_i) \\
 &= \vec{p}_f - \vec{p}_i \\
 &= \Delta \vec{p}
 \end{aligned}$$

$$\left. \begin{aligned}
 \vec{I} &= \vec{F} \cdot \Delta t \text{ — Impulse .} \\
 \vec{p} &= m\vec{v} \text{ — momentum .}
 \end{aligned} \right\} \Rightarrow \vec{I} = \Delta \vec{p}$$

$$\begin{array}{ccc}
 \vec{I} & = & \Delta \vec{p} \quad . \quad (\text{Impulse-Momentum Thm.}) \\
 \uparrow & & \uparrow \\
 \text{Impulse} & & \text{change in momentum .}
 \end{array}$$

e.g. Pile Driver

What's the average force acting on the pile?

$$v_i = 20 \text{ m/s}, \quad v_f = 0$$

$$m = 100 \text{ kg}, \quad \Delta t = 0.1 \text{ s}$$

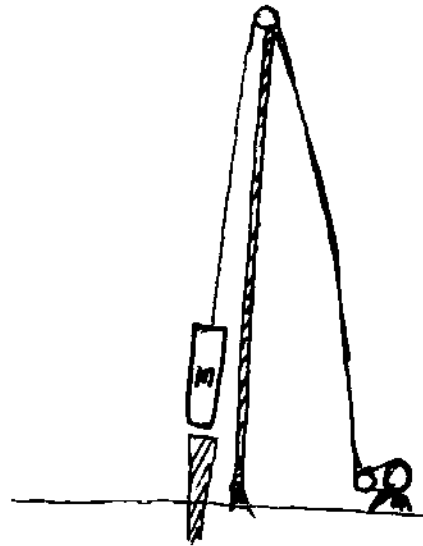
$$F = ?$$

$$F \cdot \Delta t = \Delta p =$$

$$= m \cdot \Delta v$$

$$= 100 \text{ kg} \cdot 20 \text{ m/s}$$

$$F = \frac{\Delta p}{\Delta t} = \frac{100 \text{ kg} \cdot 20 \text{ m/s}}{0.1 \text{ s}} = 20,000 \text{ N}$$



- Force acting on m through a distance Δx .

e.g., Simple case: 1-D. constant a .

$$\begin{aligned}
 F \cdot \Delta x &= F \cdot \frac{v_f^2 - v_i^2}{2a} \\
 &= ma \cdot \frac{v_f^2 - v_i^2}{2a} \\
 &= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2
 \end{aligned}
 \quad \left\{ \begin{array}{l} \Delta x = v_i t + \frac{1}{2} a t^2 \\ v_f = v_i + a t \\ \cancel{t} = \frac{v_f - v_i}{a} \\ \therefore \Delta x = \frac{v_f^2 - v_i^2}{2a} \end{array} \right.$$

i.e. $\boxed{W = K_f - K_i = \Delta K} \quad (\star)$

where $K = \frac{1}{2} m v^2$ — Kinetic Energy.

$W = F \cdot \Delta x$ — work done by force F on object m .

($\star$) — Work-Energy Theorem;

The total work done on an object is equal to the change in its kinetic energy.