Physics 101

Lecture 10. Mon. Jan. 26, 2004

Topics: Work-Energy Theorem
Kinetic Energy.
work, power.

Ref: 7-1, 2, 3, 4.

- Net force $\vec{F}$ acting on m through a distance Δx.

e.g. Simple case, 1-D. constant $\vec{a}$.

$$F\,\Delta x = F\,\frac{v_f^2 - v_i^2}{2a}$$

$$= ma\cdot\frac{v_f^2 - v_i^2}{2a}$$

$$= \frac{1}{2}m v_f^2 - \frac{1}{2}m v_i^2$$

$\Delta x = v_i t + \frac{1}{2}at^2$

$v_f = v_i + at$.

eliminate t: $\Delta x = \dfrac{v_f^2 - v_i^2}{2a}$.

i.e. $$\boxed{W = K_f - K_i = \Delta K}. \quad (\star)$$

where $K = \frac{1}{2}mv^2$ — Kinetic energy.

$W = F\,\Delta x$ — work done by $\vec{F}$, on object m.

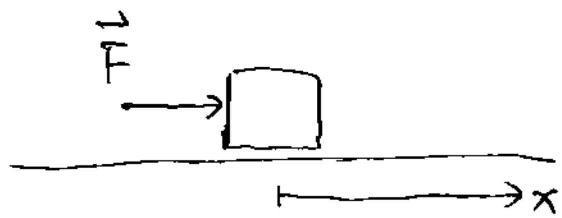
$(\star)$ — work-Energy Thm:

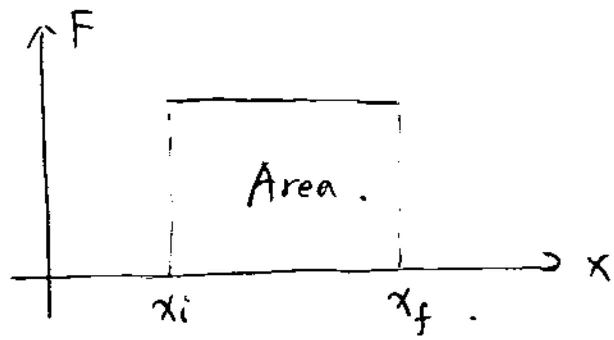
The Total work done on an object is equal to the change in kinetic energy of the object.

• Work of a force $\vec{F}$.

• Simplest case : 1-D, constant F.

$$W = F \cdot \Delta x$$



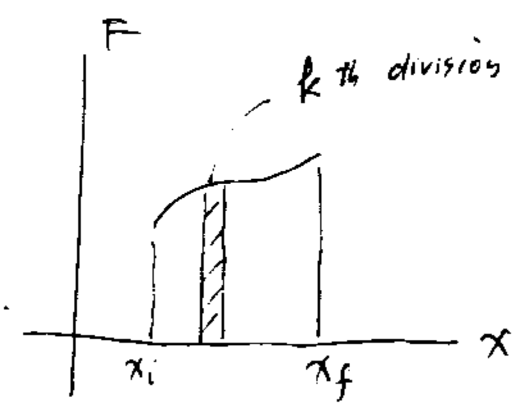


$$W = F \cdot \Delta x = \text{area}$$

• When F is NOT constant.

Idea: For each small ,

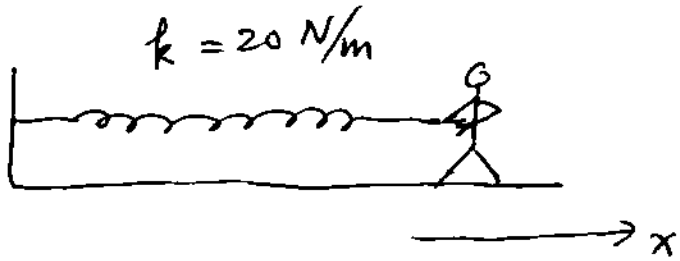
$F_k \approx$ cost, W_k = area of .



$\therefore$ Total work $W = \sum W_k$.

= Total area under curve $F(x)$.

e.g.:
1-D



How much work is needed
To stretch the spring by 0.1 m from its equilibrium.

Force of spring: $F_s = -kx$.

Force of man acting on the spring: $F_m = kx$.

$W =$ area of △

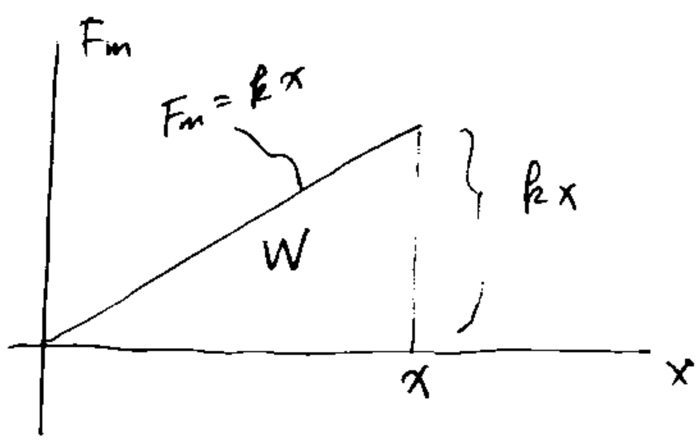
$= \frac{1}{2} \cdot x \cdot kx$

$= \frac{1}{2} kx^2$

$= \frac{1}{2}(20\ N/m) \cdot (0.1 m)^2$

$= 0.1 (N \cdot m)$

$= 0.1$ Joules.

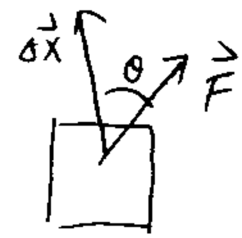


- Work of a force. (2-D or 3-D):

When $\vec{F}$ is NOT along $\Delta\vec{x}$.

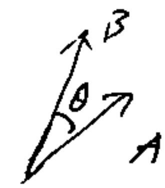
$$W = \vec{F} \cdot \Delta\vec{x}$$

$$= F \cdot \Delta x \cdot \cos\theta$$



- Dot product between 2 vectors:

$$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cdot \cos\theta$$

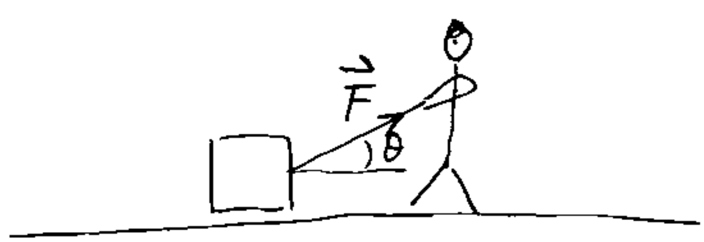


$$W = \vec{F} \cdot \Delta\vec{x} = \Delta x \cdot F \cdot \cos\theta .$$

$\Delta\vec{x}$ — displacement.

$F\cos\theta$ — Force $\underbrace{\text{component}}_{\text{(projection)}}$ along $\Delta\vec{x}$ direction.

e.g.



if $F = 40\,N$, $\theta = 30°$, moved by 2m.

How much work is needed?

$$W = (40\,N) \cdot (2m) \cdot \cos 30° = 69.3 \text{ Joules}.$$

- Power. — work per unit time.

$$P = \frac{W}{\Delta t} = \frac{\vec{F} \cdot \Delta \vec{x}}{\Delta t} = \vec{F} \cdot \vec{v}.$$

e.g.: The drag force acting on a rocket follows the quadratic rule: $F_{drag} = C_2 v^2$. When the rocket travels at 300 m/s, $F_{drag} = 900\,N$.

Q1: What is the power needed to maintain this speed?

$$\begin{aligned} P &= F \cdot v. \\ &= (900\,N)(300\,m/s) \\ &= 270{,}000\ N \cdot m/s. \\ &= 270\ kW. \end{aligned}$$

$(1\ \text{Watt} = 1\ N \cdot m/s = 1\ J/s)$.

Q2: How much power is needed when the speed is doubled?

$$F \propto v^2.$$

$$P = F \cdot v \propto v^2 \cdot v \propto v^3.$$

$\because (2v)^3 = 8v^3$. (8 times as much)

$\therefore P = 8 \times 270\ kW = 2160\ kW$.