Physics 101.

Lecture 11.

[redacted] Jan. 28. 2004, Wed.

Topics covered:

Conservative and nonconservative forces.

Potential Energy.

Generalized work-Energy Theorm.

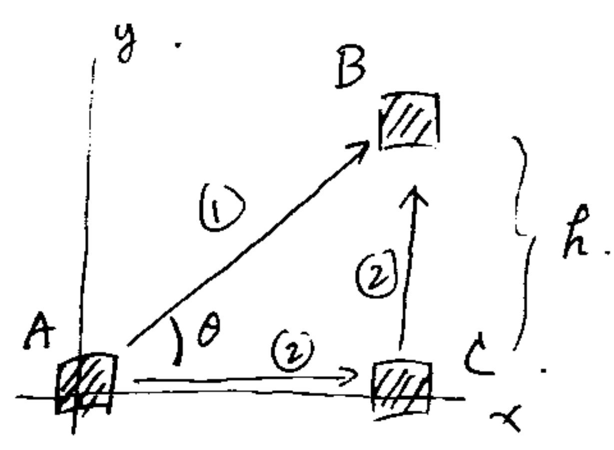
Ref: 8-1, 2, 3, 4.

- conservative force

  — Its work is independent of path

  i.e. only depends on initial and final positions.

- e.g: Gravitational force near the surface of earth.



path ①: A $\longrightarrow$ B.

Work of gravity $m\vec{g}$:

$$W_{AB} = m\vec{g} \cdot \vec{AB}$$
$$= mg \cdot |\vec{AB}| \cdot \cos(\theta + 90^\circ)$$
$$= -mg|\vec{AB}|\sin\theta$$
$$= -mgh.$$

path ②. $A \to C \to B$.

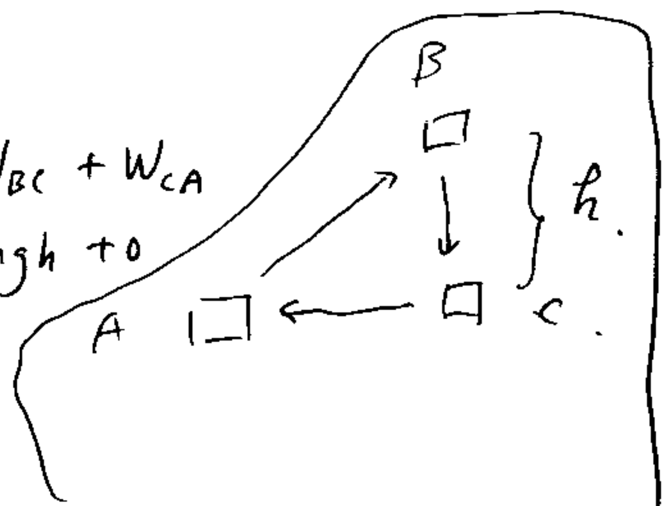
$$W_{ACB} = W_{AC} + W_{CB}$$
$$= 0 + m\vec{g} \cdot \vec{CB}$$
$$= -mgh. \quad (\vec{g} \text{ is opposite to } \vec{CB})$$

Compare the two paths:

$$W_{AB} = W_{ACB}. \quad (\text{same work!})$$

- Note: A conservative force does zero work in a closed path.

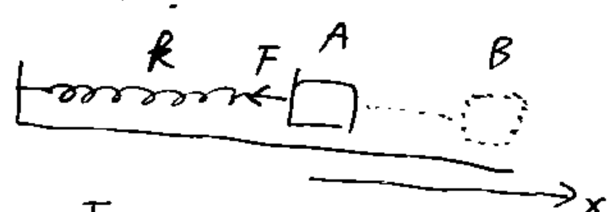
$$W_{ABCA} = W_{AB} + W_{BC} + W_{CA}$$
$$= -mgh + mgh + 0$$
$$= 0$$

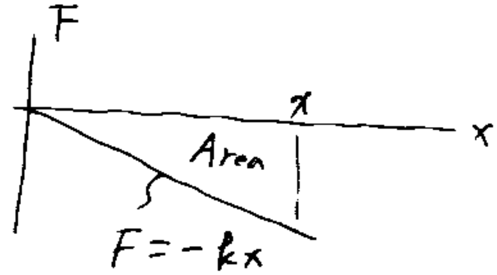


- e.g.: Force of a spring is conservative.

$$F = -kx.$$
$$W = -|\text{Area}|$$
$$= -\frac{1}{2}kx^2.$$

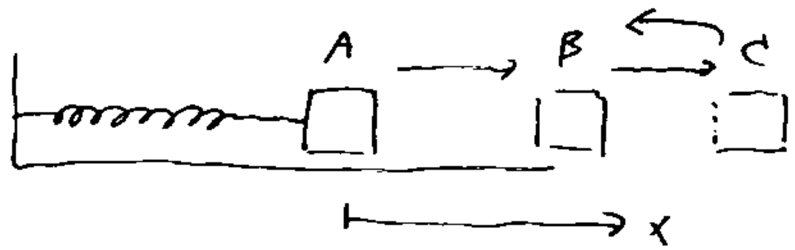
(Work done by the force of the spring).





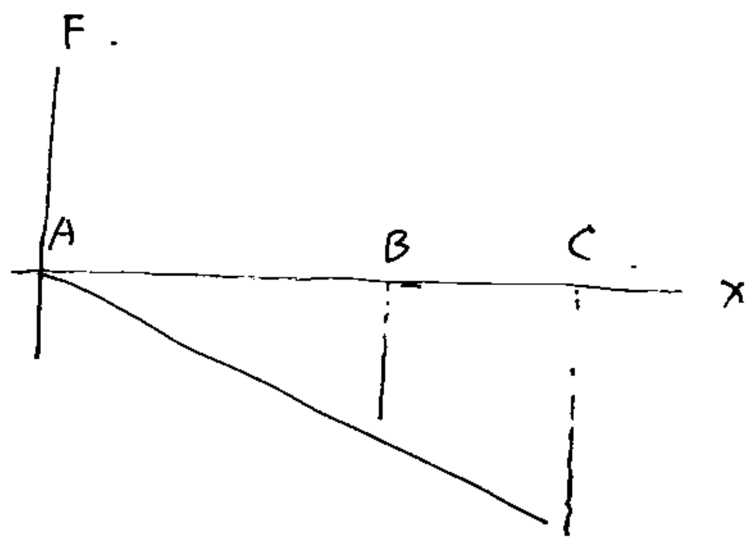
$W = W(x)$, depends on position, not path.

$W_{AB} = W_{AB} + W_{BC} + W_{CB}$.



(W_{BC} and W_{CB} has the same magnitude but opposite sign)

$\therefore\ W_{AB} = W_{ABCB}$.



- Non-conservative forces

— Work depends on path.

e.g. frictional force.

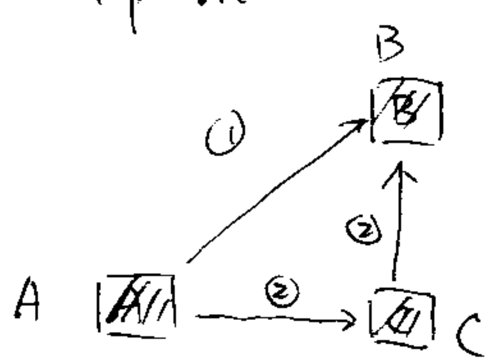
Horizontal motion

path ①: $A \to B$

path ②: $A \to C \to B$.

(No change in height!)

Top View.



path ①: work done by the frictional force.

$$W_{AB} = \vec{f} \cdot \vec{AB} = -\mu_k N |AB|$$

path ②:

$$W_{ACB} = -\mu_k N |AC| - \mu_k N |CB|.$$

$$= -\mu_k N \left(|AC| + |CB| \right)$$

In triangle ΔABC, $|AC| + |CB| \neq |AB|$.

$\therefore$ $W_{AB} \neq W_{ACB}$.

(actually, $|AC| + |CB| > |AB|$).

- Another way to view the difference between a conservative force and a nonconservative force:

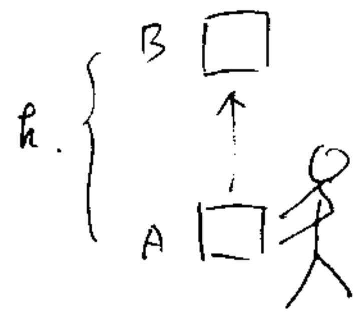
— Conservative : work can be stored as potential energy.

— nonconservative : work is dissipated into heat or sound.

- Potential Energy.

When we overcome a conservative force to do work, the work is stored as potential energy.

e.g.: The man has to overcome $m\vec{g}$ to move the box from $\underline{A \text{ to } B}$. The work he does on the box:



$$W = mgh$$

The box gains potential energy mgh. $\left(\Delta U_{A\to B} = mgh\right)$

When the object is ~~...~~ released from B, it can fall down back to A. The gravitational force does work:

$$W_c^{B\to A} = mgh. \quad \left(\text{Work of a conservative force}\right)$$

The object loses potential energy mgh. $\left(\Delta U_{B\to A} = -mgh\right)$.

$$\therefore \quad \boxed{W_c = -\Delta U} \text{ — definition of P.E.}$$

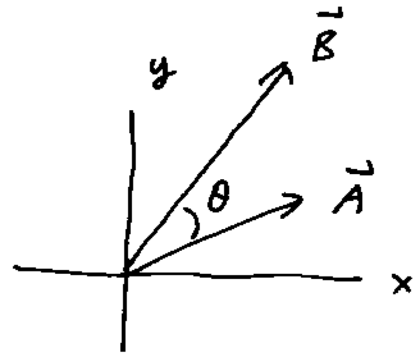
where U — potential energy.

ΔU — change in potential energy.

$-\Delta U$ — P.E. lost.

- Dot product using components.

$$\vec{A}\cdot\vec{B} = A\ B\cos\theta$$



$$\vec{A} = A_x\,\vec{i} + A_y\,\vec{j}$$

$$\vec{B} = B_x\,\vec{i} + B_y\,\vec{j}$$

$$\vec{A}\cdot\vec{B} = (A_x\,\vec{i} + A_y\,\vec{j})\cdot(B_x\,\vec{i} + B_y\,\vec{j})$$

$$= A_xB_x\,\vec{i}\cdot\vec{i} + A_xB_y\,\vec{i}\cdot\vec{j} + A_yB_x\,\vec{j}\cdot\vec{i} + A_yB_y\,\vec{j}\cdot\vec{j}$$

$$= A_xB_x + A_yB_y\,.$$

$$\left[\begin{array}{l} \vec{i}\cdot\vec{i} = \vec{j}\cdot\vec{j} = 1\,. \quad (\cos 0^\circ = 1) \\ \vec{i}\cdot\vec{j} = \vec{j}\cdot\vec{i} = 0\,. \quad (\cos 90^\circ = 0) \end{array}\right.$$

In general, 3-$\vec{D}$:

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z\,.$$