Physics 101

Lecture 12

Friday, Jan. 30, 2004

Topics: Potential Energy

e.g.: Spring.

Gravity

Generalized work-energy theorem

Conservation of Mechanical Energy.

Ref: 8-1, 2, 3, 4

Note: When you use the conservation Laws, Watch out the conditions!

e.g.: what's the condition for Total momentum to be conserved?

$$\vec{F}_{ext} = 0$$

- The potential energy is defined as:

  The work done by a conservative force is equal to the decrease in potential energy. $W_c = -\Delta U$ or $\Delta U = -W_c$

Note: We only have defined the change in potential energy, i.e., ΔU, but NOT U itself.

Therefore, ~~we can~~ when we define/use potential energy U, we can ~~arbitrary~~ choose the zero point of P.E. at our convenience.

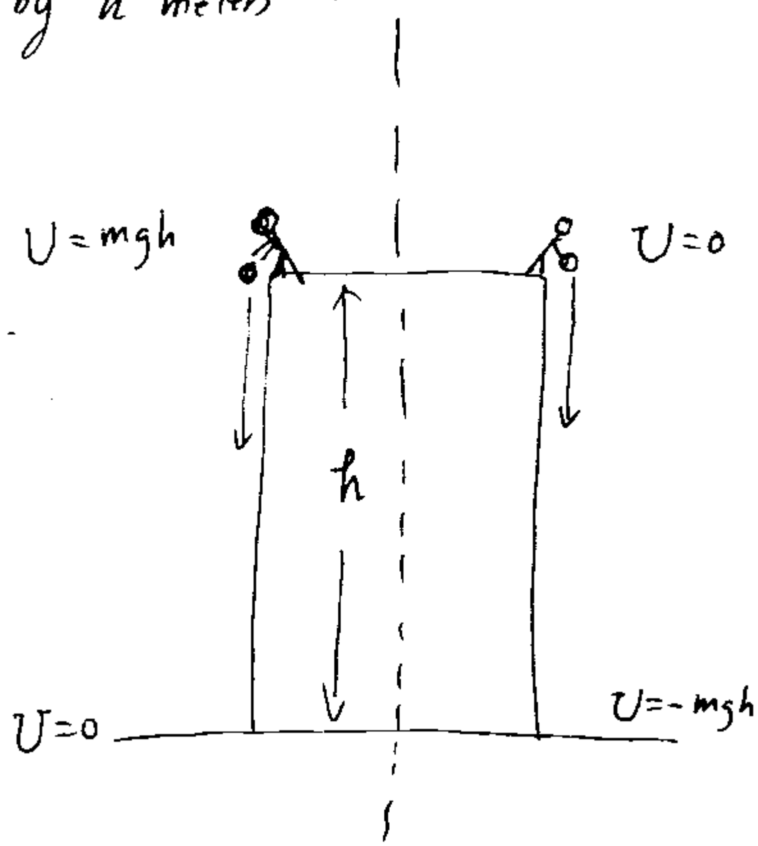
e.g.: A ball has fallen by h meters.

We can choose
$U=0$ ~~at~~ the ground (left)
OR on the top of building (right).

No matter where we choose the zero point of P.E,
we always have:

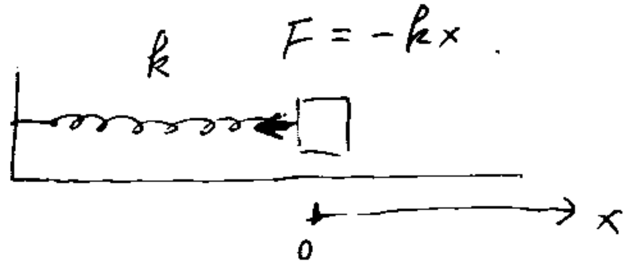
$$\Delta U = U_f - U_i$$
$$= -mgh.$$

$$\begin{pmatrix} W_c = mgh \\ \Delta U = -W_c \end{pmatrix}$$

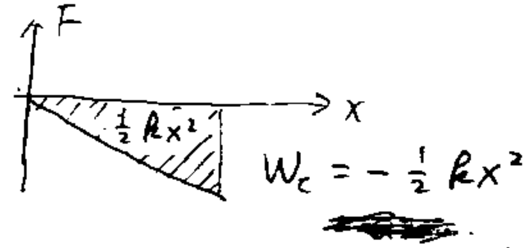


- Potential Energy of a Spring.

$$\Delta U = -W_c$$



Choose $U=0$ at $x=0$.
(equilibrium)

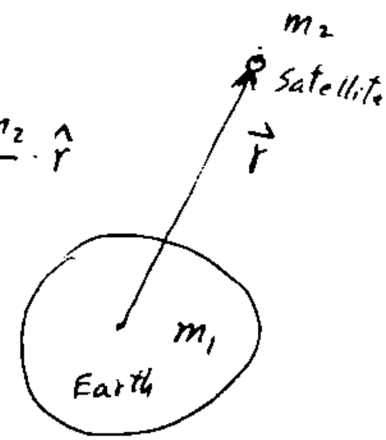


$\therefore\ U = -(-\frac{1}{2}kx^2)$

i.e., $U(x) = \frac{1}{2}kx^2$.

when we stretch the spring (from $x=0$ to x meters) by x meters, we raise it's potential energy by $\frac{1}{2}kx^2$.

- Potential Energy of Gravity



Force acting on m_2: $\vec{F} = -G\frac{m_1 m_2}{r^2}\cdot\hat{r}$

choose $U=0$ at $r=\infty$

Can be shown: $U(r) = -G\frac{m_1 m_2}{r}$

Near the surface of earth, if r is changed from R_E to R_E+h:
(object lifted by h)

$$\Delta U = \left(-G\frac{m_1 m_2}{R_E+h}\right) - \left(-G\frac{m_1 m_2}{R_E}\right) = G\frac{m_1}{R_E^2}\cdot m_2 h.$$

$\therefore\ \Delta U = mgh.$

$\left(\begin{array}{l} g = G\cdot\frac{m_1}{R_E^2} \\ m_1 \text{ — mass of the earth} \\ m = m_2 \text{ — mass of the satellite} \end{array}\right)$

• Generalized Work-Energy Theorm.

Forces are divided into Two kinds $\begin{cases} \text{conservative} \\ \text{nonconservative} \end{cases}$.

Then The Total work on an object:

$$W = W_c + W_{nc}.$$

but, the conservative work is related to P.E:

$$W_c = -\Delta U.$$

Therefore, the work-Energy Thm $W = \Delta K$ becomes:

$$W = \Delta K.$$

$$W_c + W_{nc} = \Delta K.$$

$$-\Delta U + W_{nc} = \Delta K.$$

$$W_{nc} = \Delta K + \Delta U = \Delta(K+U) = \Delta E.$$

where $E = K + U$ is called Mechanical Energy.

∴ Generalized work energy Theorm:

$$\boxed{W_{nc} = \Delta E}$$

{Nonconservative work on an object.} = {change in mechanical energy of the object}.

- When $W_{nc} = 0$ (i.e., No nonconservative work)

$$\Delta E = 0 . \quad (\because W_{nc} = \Delta E .)$$

i.e.: $E = \text{const.}$
OR: $E_f = E_i$.
} Mechanical Energy is conserved.

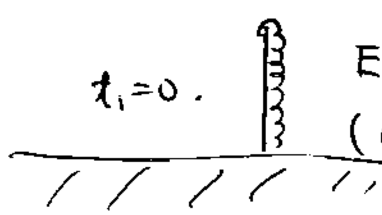
$\therefore$ The condition for Mechanical Energy conservation is there is no nonconservative work.
($W_{nc} = 0$)

Demo: Spring.

U_k = P.E. of spring.
U_g = P.E. of Gravity.
K = kinetic energy.

t_3  $h = \max$.
$E_3 = U_g$
$K = 0$, $U_k = 0$.

t_2  $E_2 = U_g + K$
($U_k = 0$)

$t_1 = 0$.  $E_1 = U_k$
($U_g = 0$, $K = 0$)

$$E_1 = E_2 = E_3 .$$

(Mechanical Energy is conserved)

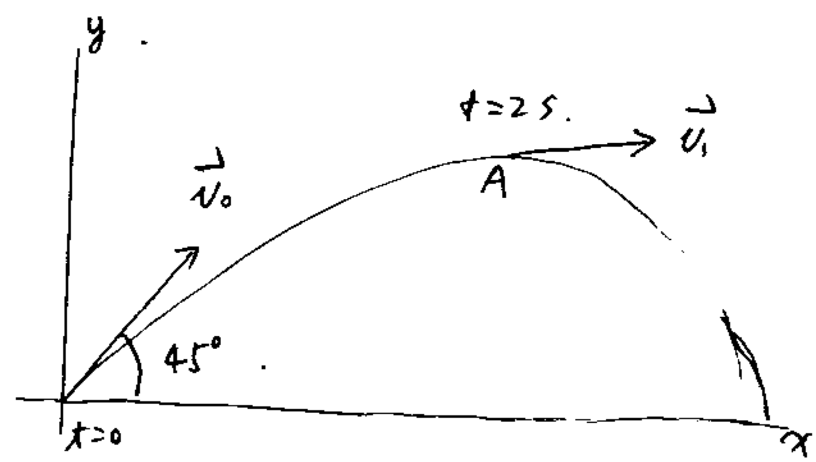
$\because W_{nc} = 0$.
(ignore air resistance).

e.g.: Phys 101. Final Exam. (Fall 2002)

Q# 16.

a). $\vec{a}_{av} = ?$

b). $W_{Drag} = ?$



a). Air resistance can not be ignored!

$\therefore$ Not a typical projectile motion $\begin{cases} x = x_0 + v_{0x} t \\ y = y_0 + v_{0y} t - \frac{1}{2} g t^2 \end{cases}$

How to find $\vec{a}_{av}$:

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}. \qquad \Delta \vec{v} = \vec{v}_f - \vec{v}_i \text{ . (Vector subtraction).}$$

use the component method: $a_x = \frac{\Delta v_x}{\Delta t}$.

$$a_y = \frac{\Delta v_y}{\Delta t}.$$

Then $a = \sqrt{a_x^2 + a_y^2}$.

$\theta = \tan^{-1} \frac{a_y}{a_x}$. (when $a_x < 0$, and $a_y < 0$. θ is in Quadrant III. $180° < \theta < 270°$.)

b). Work done by the Drag.

Drag — nonconservative force.

delaits — complicated, time-dependent.

Idea: use the generalized Work-Energy Theorm which only needs the initial and final Kinetic Energy.

$$W_{nc} = \Delta E = \Delta K + \Delta U$$

$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 + mgh.$$

For detailed solution, please visit the course web site. http://www.sfu.ca/~mxchen/