

Physics 101

Lecture 13

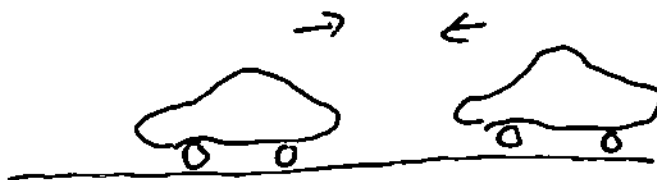
Monday, Feb. 2, 2004

Topics: Collisions $\left\{ \begin{array}{l} \text{elastic} \\ \text{inelastic} \\ \text{completely inelastic} \end{array} \right.$

Ref: 9-1, 2, 3, 4, 5, 6, 7.

Demo: Collisions

• Collision



Typically: Time of interaction — very short.

Force of impact — complicated.

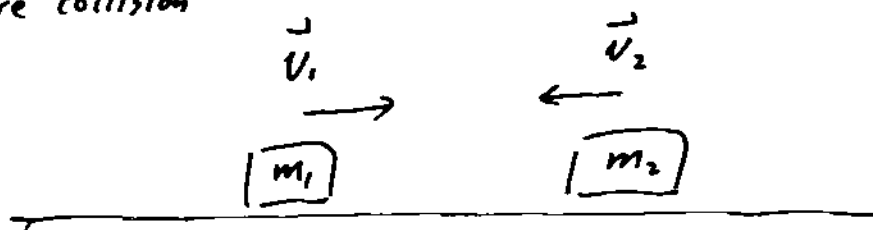
much stronger than gravity
much stronger friction.

Idea: Focus on the states of Motion before and after
the collision

Use the theorems $\left\{ \begin{array}{l} \vec{I} = \Delta \vec{p} \\ W = \Delta K \end{array} \right\}$ applied to the 2 bodies

- Notation

before collision



After collision $\vec{v}_1'$ $\vec{v}_2'$

- Newton's law applied to each object:

$$\Sigma \vec{F}_1 = \frac{\Delta \vec{p}_1}{\Delta t} \quad (\Sigma \vec{F}_1 = \text{Net force acting on } m_1)$$

$$\Sigma \vec{F}_2 = \frac{\Delta \vec{p}_2}{\Delta t} \quad (\Sigma \vec{F}_2 = \text{Net force acting on } m_2)$$

Then:

$$\Sigma \vec{F}_1 + \Sigma \vec{F}_2 = \frac{\Delta \vec{p}_1 + \Delta \vec{p}_2}{\Delta t}$$

i.e.:

$$\boxed{\vec{F} = \frac{\Delta \vec{P}}{\Delta t}} \quad (1)$$

where

$\vec{F}$ = Total force = sum of all forces acting on m_1 and m_2 .

$$\vec{P} = \vec{p}_1 + \vec{p}_2 = \text{Total momentum}$$

- Because of Newton's 3rd law, $\vec{F}_{12} = -\vec{F}_{21}$,

all the internal forces in (1) cancel!

Therefore, (1) becomes:

$$\boxed{\vec{F}_{\text{ext}} = \frac{\Delta \vec{p}}{\Delta t}} \quad (2)$$

i.e., only the external forces contribute to the change in total momentum.

- Conservation of Total momentum.

$$\text{In (2), when } \vec{F}_{\text{ext}} = 0, \quad (3)$$

$$\Delta \vec{p} = 0.$$

$$\text{i.e., } \vec{p} = \text{const.}$$

When the net external force is zero,
the total momentum is conserved.

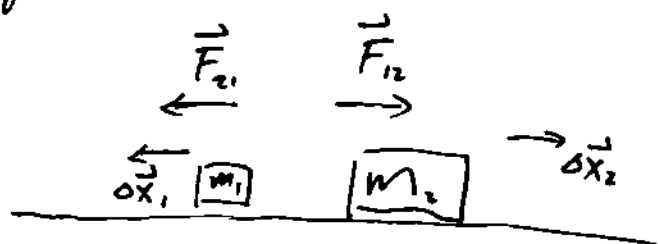
Note: eq. (3) is a vector equation. We can use
a component of (3) if a component of $\vec{F}_{\text{ext}}$ is zero.

- In a collision problem, often $\vec{F}_{\text{ext}} \approx 0$.

Then, $m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_1' + m_2 \vec{v}_2'$

i.e.: $\vec{P}_i = \vec{P}_f$ (conservation of total momentum)

- Work by internal forces won't cancel!



$$W_1 = \vec{F}_{21} \cdot \delta \vec{x}_1 \quad (\text{work of internal force } \vec{F}_{21})$$

$$W_2 = \vec{F}_{12} \cdot \delta \vec{x}_2 \quad (\text{work of internal force } \vec{F}_{12})$$

Although $\vec{F}_{12} = -\vec{F}_{21}$,

$\delta \vec{x}_1$ can have an opposite sign as $\delta \vec{x}_2$ as well.

i.e., both W_1 and W_2 can be positive at the same time.

$\therefore$ W_1 and W_2 don't ~~cancel~~ cancel!

i.e., internal forces can do net work on the system.

• Kinetic Energy .

$$W_1 = \Delta K_1 .$$

$$W_2 = \Delta K_2 .$$

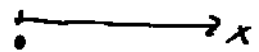
$$W_1 + W_2 = \Delta (K_1 + K_2) = \Delta K .$$

Even when the net external force is zero,
the total kinetic energy is not necessarily zero
because the internal work can contribute .

e.g: 2 Astronauts playing catch .

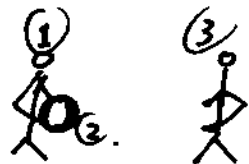
(A system of 3 objects) . $\vec{P}_T = \vec{P}_1 + \vec{P}_2 + \vec{P}_3$.

$$1-D: P_T = P_1 + P_2 + P_3 .$$



a) : $P_1 = 0, P_2 = 0, P_3 = 0$.

$$P_T = 0 .$$

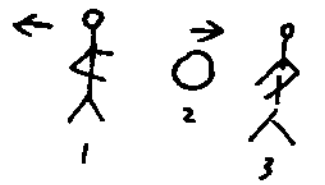


b) . $P_2 = 10 \text{ kg} \cdot \text{m/s}$.

$$P_1 = -10 \text{ kg} \cdot \text{m/s} .$$

$$P_3 = 0 .$$

$$P_T = 0 . \quad K_T \neq 0 = K_1 + K_2$$



c) . $P_1 = -10, P_2 + P_3 = 10$.

$$P_T = 0 . \quad K_T \neq 0 .$$



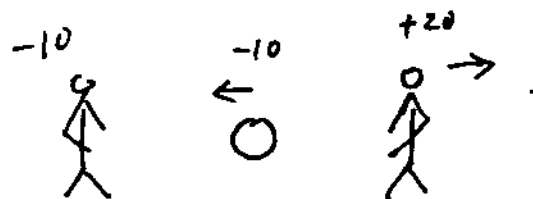
$$d) \quad P_1 = -10 \quad .$$

$$P_2 = -10 \quad .$$

$$P_3 = 20 \quad .$$

$$P_T = 0 \quad .$$

$$K_T \neq 0 \quad .$$



It's clear that $P_T = 0$ ~~always~~ — Not changed :
 $\therefore$ No external force.

but. $K_T \neq 0$ — ~~has~~ increased each time.
 $\therefore$ internal work.

(K_1, K_2, K_3 won't cancel
 because $\frac{1}{2}mv^2$ is always positive)

$\therefore$ Internal forces can change the total kinetic energy
 of the system, but can not change the total momentum.

- Conservation of Total kinetic energy .

— When the collision is elastic

↑ Hit and bounce .

(No deformation)

(No energy loss)

$$\Delta K = 0, \quad K_i = K_f.$$

$$\text{i.e.} \quad \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

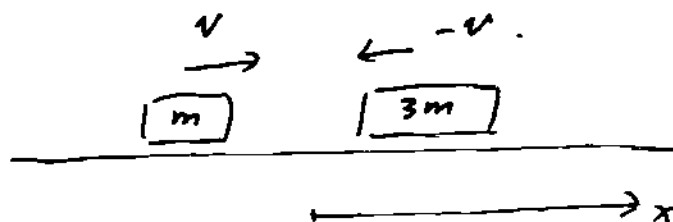
- Inelastic collision — K is not conserved .

- Completely inelastic collision — Hit and stick .

— K is not conserved .

but $\vec{v}_1' = \vec{v}_2'$, i.e., the two objects
move together after the
collision .

e.g.: Fall 2002 final exam #17.



It's a 1-D collision.

Set up the coordinate system first.

— Net external force: $\vec{F}_{\text{ext}} = 0$.

$\therefore$ Total Momentum is conserved:

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_1' + m_2 \vec{v}_2'$$

$$\text{i.e.: } mV - 3mV = mV_1' + 3mV_2'$$

$$\text{OR: } -2V = V_1' + 3V_2' \quad (1)$$

— Since it's elastic collision, K.E. is conserved:

$$\frac{1}{2}mV^2 + \frac{1}{2}(3m)V^2 = \frac{1}{2}mV_1'^2 + \frac{1}{2}(3m)V_2'^2$$

$$\text{i.e.: } 4V^2 = V_1'^2 + 3V_2'^2 \quad (2)$$

Solve (1) and (2) for V_1' and V_2' .

$$\text{solution 1. } \begin{cases} V_1' = -2V \\ V_2' = 0 \end{cases}$$

m_1 moves back at $-2V$.
 m_2 stops.

$$\text{solution 2: } \begin{cases} V_1' = V \\ V_2' = -V \end{cases}$$

No collision — rejected.

What if the collision is completely inelastic?

Total momentum is still conserved, but the total kinetic energy is no longer conserved.

ie. we still have ① : $-2V = V_1' + 3V_2'$ ①

but eq. ② is NOT valid any more.

We need another equation to solve for V_1' and V_2' .

— completely inelastic : $V_1' = V_2'$ ③.

~~That~~ That helps!

subs ③ into ① : $-2V = 4V_2'$

$$V_2 = -\frac{1}{2}V = V_1.$$

∴ both m_1 and m_2 will move at a slower speed.

$$(V_1 = V_2 = -\frac{1}{2}V)$$