Physics 101.

Lecture 14 Wed. Feb. 4, 2004.

Topics : Centre of Mass

Examples.

Demo : Collision

Ref : 9-7, 10-1, 2, 3, 4, 5,

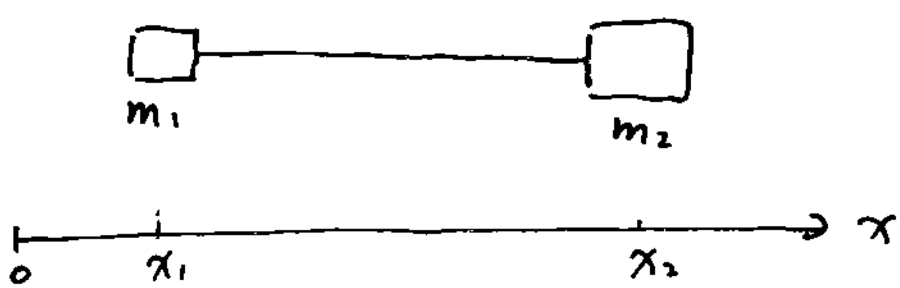
- Centre of Mass (C.M.)

— of Many-particle systems

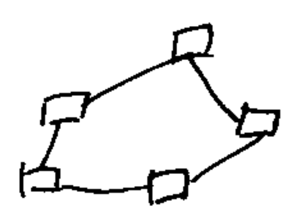
OR extended objects

C.M. — A point to represent the "average" position

e.g:



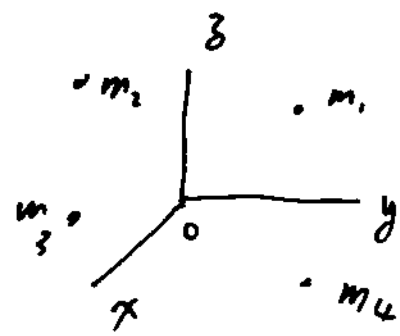
$$x_c = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$



$$x_c = \frac{\sum m_i x_i}{M}, \qquad M = \sum m_i \text{ — Total mass}$$

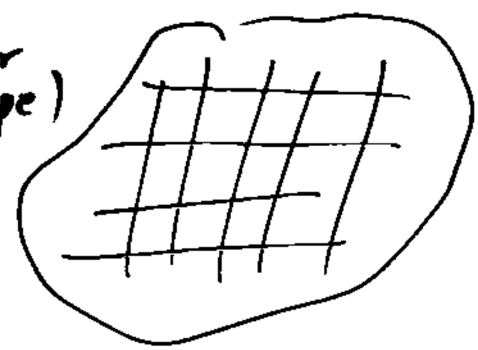
- Many-Particle system in 3-D.

$$\left.\begin{aligned} x_c &= \frac{\sum m_i x_i}{M} \\ y_c & \\ z_c & \end{aligned}\right\} \quad \vec{r}_c = \frac{\sum m_i \vec{r}_i}{M}$$



- Extended object.

Idea: Divide it into many small pieces (regular shape)



still: $\vec{r}_c = \dfrac{\sum m_i \vec{r}_i}{M}$.

$\vec{r}_i$ — C.M. of ith piece.

• Potential Energy.

Total P.E. of System = P.E. of C.M.

Why?

e.g.: $$y_c = \frac{\Sigma m_i y_i}{M}.$$

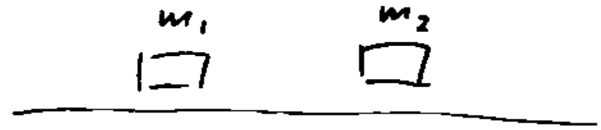
$$M y_c = \Sigma m_i y_i$$

$$M g y_c = \Sigma m_i g y_i$$

i.e. P.E. of. C.M. = Total P.E.

- Momentum.

Total momentum of m_1 and m_2



$$\begin{aligned}
\vec{P} &= \vec{p}_1 + \vec{p}_2 \\
&= m_1\vec{v}_1 + m_2 V_2 \\
&= m_1 \frac{\Delta\vec{r}_1}{\Delta t} + m_2 \frac{\Delta\vec{r}_2}{\Delta t}. \\
&= \frac{M}{M} \cdot \frac{1}{\Delta t} \cdot \left(m_1 \Delta\vec{r}_1 + m_2 \Delta\vec{r}_2 \right) \\
&= \frac{M}{\Delta t} \cdot \frac{m_1 \Delta\vec{r}_1 + m_2 \Delta\vec{r}_2}{M} \\
&= \frac{M}{\Delta t} \cdot \Delta\vec{r}_c \\
&= M \cdot \vec{V}_c. \\
&= \vec{P}_c.
\end{aligned}$$

$\therefore$ Total momentum = Momentum of C.M.

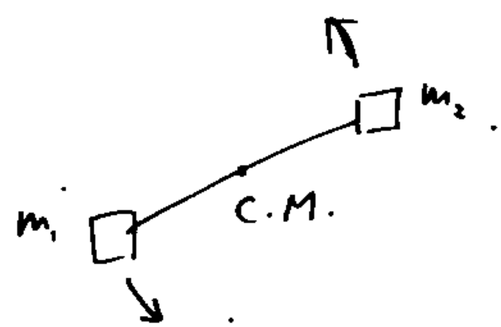
- Kinetic Energy.

However, $K_{Total} \neq K_c$.

i.e. $K_{Total} \neq \frac{1}{2} M \cdot V_c^2$.

The total kinetic energy ~~does~~ is not equal to the kinetic energy of C.M.

e.g:

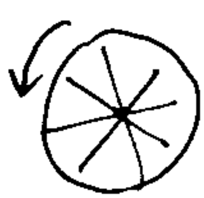


$V_c = 0$. but $V_1 \neq 0$, $V_2 \neq 0$.

$$K_c = \frac{1}{2} M V_c^2 = 0.$$

but. $$K_{Total} = K_1 + K_2 = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2 \neq 0.$$

e.g: A rotating wheel with a fixed axis.



C.M. is not moving: $V_c = 0$.

but there is a rotational kinetic energy.

e.g: CAPA set #3.
Question # 11.

Different ways, but the same idea:
Mechanical Energy is conserved.
$\because W_{nc} = 0$.

see. solutions for CAPA set #3 on the web.

e.g. CAPA set #3. Question # 12. } 3 stages: Free falling.
Collision
Free "falling" again.

{Key ideas for collision}: conservation of Momentum — $\because F_{ext} \approx 0$
conservation of K.E. — $\because W_{nc} = 0$.

Note: when 2 objects collide,
Don't assume one of them will always stop!