Physics 101
Lecture 15

Friday, Feb. 6, 2004.

Topics: Rotational Kinematics
Linear and Rotational Quantities
Rolling Motion

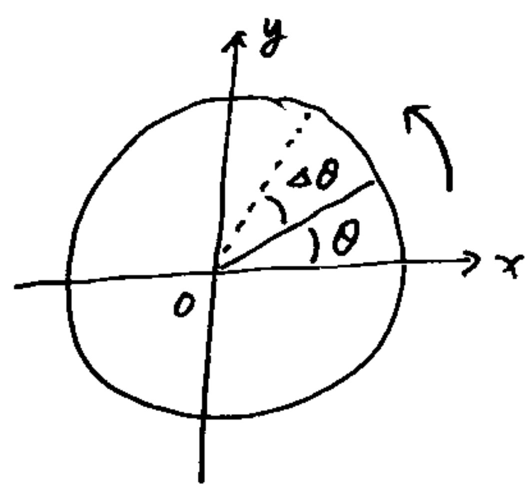
Ref: 10-1, 2, 3, 4, 5, 6.

Note: capa set #4 : Due. Feb. 19. (Thur).

- Rotational Kinematics

angular position : θ

angular displacement : $\Delta\theta$



$\Delta\theta$: + . c.c.w. $\circlearrowleft$ (R.H. out) $\odot$

− cw $\circlearrowright$ $\otimes$ (R.H. in)

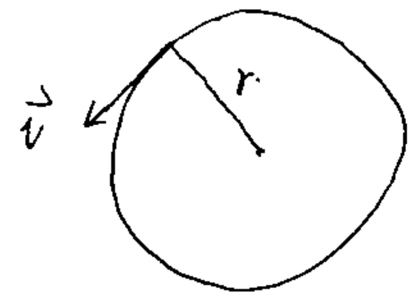
angular velocity : $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$

angular acceleration : $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$

- Linear velocity $\vec{v}$

Direction : tangential.

magnitude :

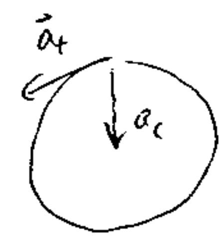


$$v = \frac{\Delta s}{\Delta t} = \frac{r \cdot \Delta\theta}{\Delta t} = r \cdot \omega$$

(as $\Delta t \to 0$)

- Acceleration $\vec{a}$. consists of two parts. (Two components).

<1>. Centripetal acceleration :



$$a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r \cdot \omega^2$$

direction : centre-seeking.

(changes the direction of $\vec{v}$)

<2>. tangential acceleration.

$$a_t = \frac{\Delta v}{\Delta t} = \frac{\Delta (r\omega)}{\Delta t} = r \cdot \frac{\Delta\omega}{\Delta t} = r \cdot \alpha \quad (\Delta t \to 0)$$

direction : tangential

(changes the magnitude of $\vec{v}$. (speed)).

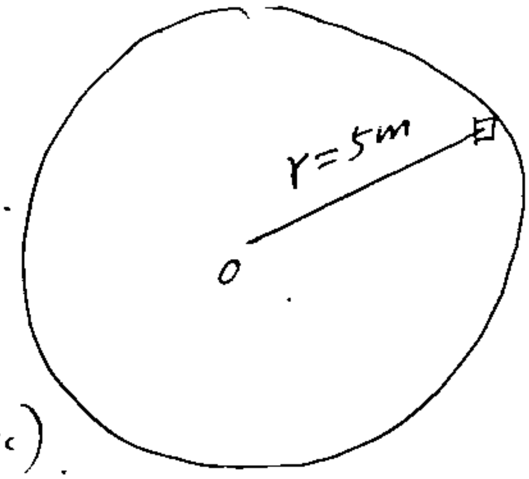
Total acceleration : $\vec{a} = \vec{a}_c + \vec{a}_t$

e.g.: Carousel $r = 5\,m$

given: $t = 0$. rest.

$t = 10\,s$. 1 rev. per 5 sec. (ω)

$\alpha = \text{const}$.



Question: $a_t = ?$ (tangential acc).

[Solution]: $a_t = r \cdot \alpha$. — We need α.

for constant α: $\alpha = \alpha_{av} = \dfrac{\Delta \omega}{\Delta t} = \dfrac{\omega_f - \omega_i}{\Delta t}$.

now: $\Delta t = 10\ (s)$.

$\omega_i = 0$.

$\omega_f = 1\ \text{rev}/5\,\text{sec} = \dfrac{1\ \text{rev}}{5\,\text{sec}} \cdot \dfrac{2\pi\ (\text{rad})}{1\,\text{rev}} = \dfrac{2\pi}{5}\ \text{rad}/\text{sec}$.

$\therefore\ \alpha = \dfrac{\frac{2\pi}{5} - 0}{10} = \dfrac{\pi}{25}\ \text{rad}/s^2$.

$a_t = r \cdot \alpha = 5 \cdot \dfrac{\pi}{25} = \dfrac{\pi}{5} = 0.63\ m/s^2$.

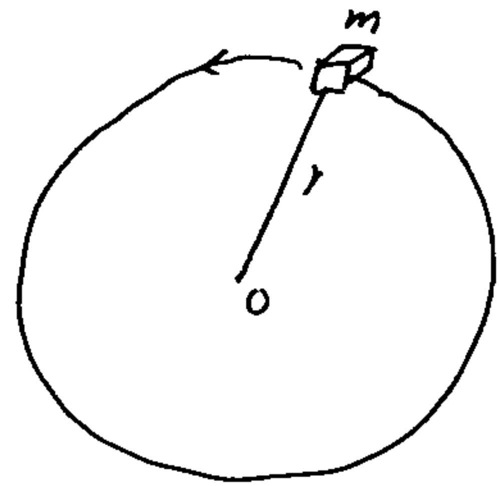
$= \text{const}$.

Note: a is Not a constant.

$a = \sqrt{a_c^2 + a_t^2}$. where $a_c = \dfrac{v^2}{r} = r \cdot \omega^2$.

Since ω is varying, a_c and a must be varying.

- Rotational Kinetic Energy and Moment of Inertia



$$K = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m r^2 \omega^2$$

$$= \frac{1}{2} I \omega^2$$

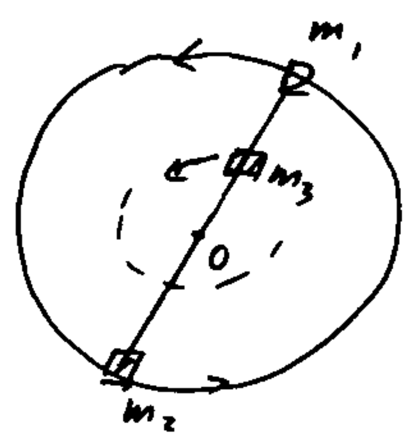
where $I = m r^2$ called

Moment of Inertia.

(Similar to mass in linear motion)

$K = \frac{1}{2} m v^2$

- Many-body:



$$K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2$$

$$= \frac{1}{2} m_1 r_1^2 w_1^2 + \frac{1}{2} m r_2^2 w_2^2 + \frac{1}{2} m r_3^2 w_3^2$$

$$= \frac{1}{2} (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2) \omega^2$$

$$= \frac{1}{2} I \omega^2$$

$w_1 = w_2 = w_3 = \omega$.

common angular velocity

$I = \sum m_i r_i^2$ — Total moment of inertia.

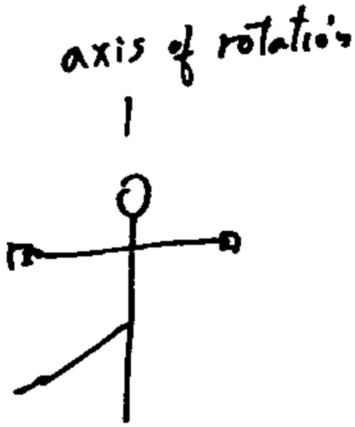
- How mass distribution affects the total moment of inertia.

$$I = \sum m_i r_i^2 .$$

To increase I, make m_i farther away from the axis of rotation.

To reduce I, put m_i close to the axis of rotation.

e.g.: Figure skating



Large I.

Small I.