Physics 101

Lecture 16

Monday, Feb. 9, 2004

Topics : Rotational Kinetic Energy
Moment of inertia
Torque
Rotational Dynamics

Ref: 10-5, 6; 11-1, 2, 3, 4, 5.

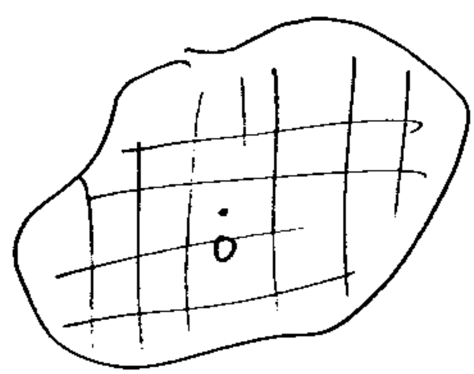
- Rotational Kinetic Energy of a rigid body.

$$K_R = \frac{1}{2} I \omega^2$$

$$I = \Sigma m_i r_i^2$$ — Moment of inertia

- Moment of inertia I

— depends on the shape of the object and the position of the axis.



O — axis of rotation

$I_{hoop} > I_{disk} = I_{cylinder} > I_{sphere}$.

See Table 10-1. on page 291. $\Rightarrow$ put them on your formula sheet!

- calculate

$$\left.\begin{aligned} I_{hoop} &= MR^2 \\ I_{disk} &= \tfrac{1}{2}MR^2 \\ I_{sphere\,(solid)} &= \tfrac{2}{5}MR^2 \end{aligned}\right\}$$

How to derive?

In general, we need calculus.

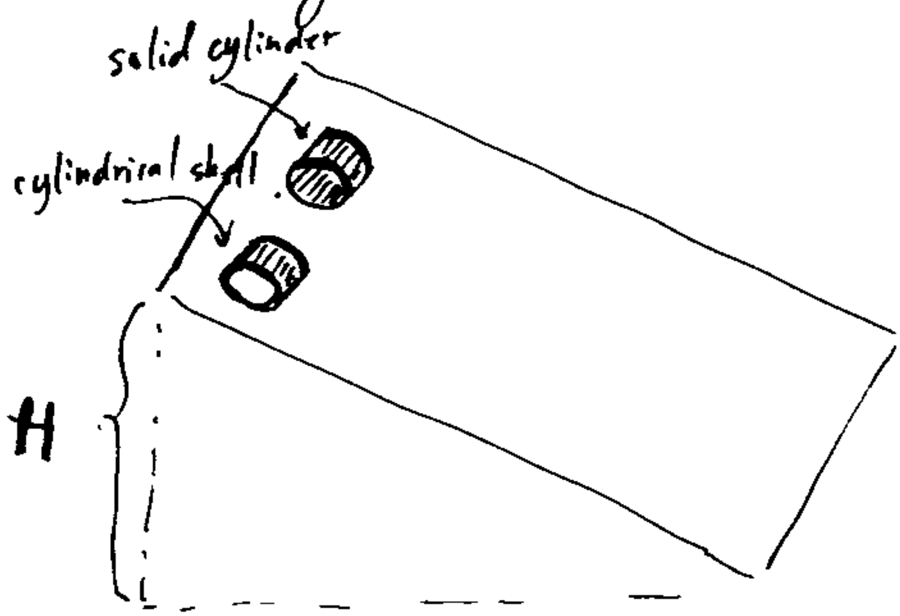
I represents the natural resistance of the object against rotational motion.

- Total kinetic Energy

$$\boxed{K = \frac{1}{2}MV_c^2 + \frac{1}{2}I\omega^2}$$

$$= \underset{\substack{\uparrow \\ \text{Linear K.E.} \\ \text{of C.M.}}}{K_L} + \underset{\substack{\uparrow \\ \text{Rotational K.E.} \\ \text{about C.M.}}}{K_R}.$$

e.g: Rolling down on an inclined plane.



Q: Which one reaches the bottom first?

No Nonconservative work. — M.E. is conserved.

~~When an object is~~

For an object rolling dowin by heigh h.

$$E_i = E_f.$$

$$mgh = \frac{1}{2}mv_c^2 + \frac{1}{2}I\omega^2.$$

but $v_c = v = r\cdot\omega$. (rolling). $\omega = \frac{v}{r}$

$$\therefore \quad mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\cdot\frac{v^2}{r^2}.$$

$$2mgh = mv^2\left(1 + \frac{I}{mr^2}\right).$$

$$v = \sqrt{\frac{2gh}{1 + \frac{I}{mr^2}}}$$

[Note: for pure sliding without friction : $v = \sqrt{2gh}$]

- v depends on $\frac{I}{mr^2}$.

  — The smaller the $\frac{I}{mr^2}$, the faster the v.

- $\frac{I}{mr^2}$ depends on the shape, Not the mass and size.

  — Solid cyliner : $I = \frac{1}{2}mr^2$, $\frac{I}{mr^2} = \frac{1}{2}$.

  — Hollow cylinder : $I = mr^2$. $\frac{I}{mr^2} = 1$. $\Rightarrow$ slower.

$\therefore$ the solid cylinder reaches the bottom first.

- Conceptual reasoning.

  $$P.E. = mgh \xrightarrow[\text{converted to}]{} K.E. = K_L + K_R = \text{constant}.$$

Large $I \Rightarrow$ Large K_R, $\Rightarrow$ small K_L $\Rightarrow$ small v.

Now, qualitatively, more mass is distributed near the axis in a solid cylinder than a shell, i.e.,

~~The moment of~~

i.e, a solid cylinder has a smaller I than a shell.
Therefore, its v is faster and it reaches the bottom first.

e.g: Compare the rolling of a solid cylinder and a solid sphere, which one reaches the bottom first?

$\because$ A solid sphere has more mass near the axis of rotation, its I is smaller. Then, its v is faster.

$\therefore$ The solid sphere reaches the bottom first.