Physics 101

Lecture 18, Wed, Feb. 18, 2004

Topics: Rotational Dynamics
Static Equilibrium of a Rigid Body
Angular Momentum
* ~~Rotational Work~~.

Ref: 11-1, 2, 3, 4, 5, 6, 7, 8

- Rotational Dynamics

$$\tau = I\alpha$$

$$\tau \text{ — net torque} = \Sigma \tau_i \quad \begin{cases} + \text{ — c.c.w} \circlearrowleft \\ - \text{ — c.w} \circlearrowright \end{cases}$$

e.g. Atwood's Machine

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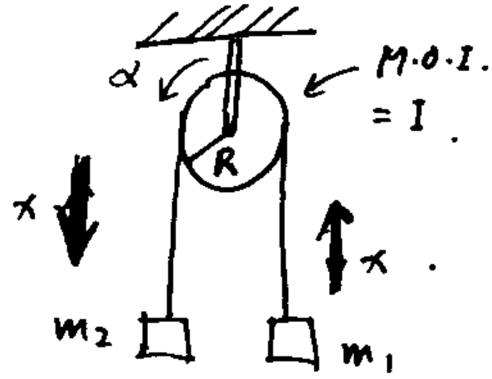
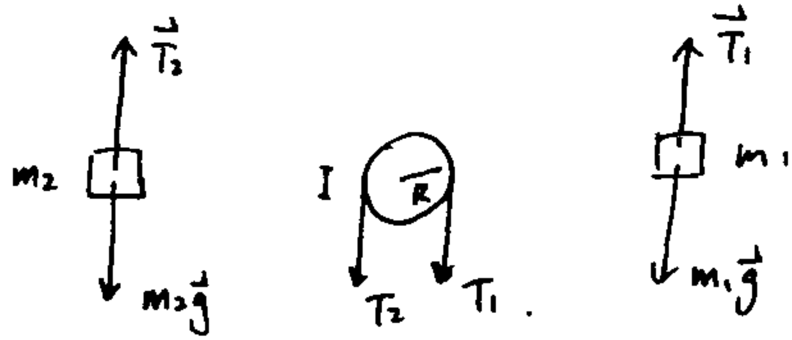
A string with a mass at each end runs without slipping over a pulley.





Typically.
given: m_1, m_2, R, I.
find: T_1 T_2, a, α.

$\cancel{T_2 - m_2 g = m_2 a}$.

$m_2 g - T_2 = m_2 a$. — ①

$T_1 - m_1 g = m_1 a$. — ②

$T_2 R - T_1 R = I\alpha$. — ③

kinematics relation: $a = \alpha \cdot r$. ④

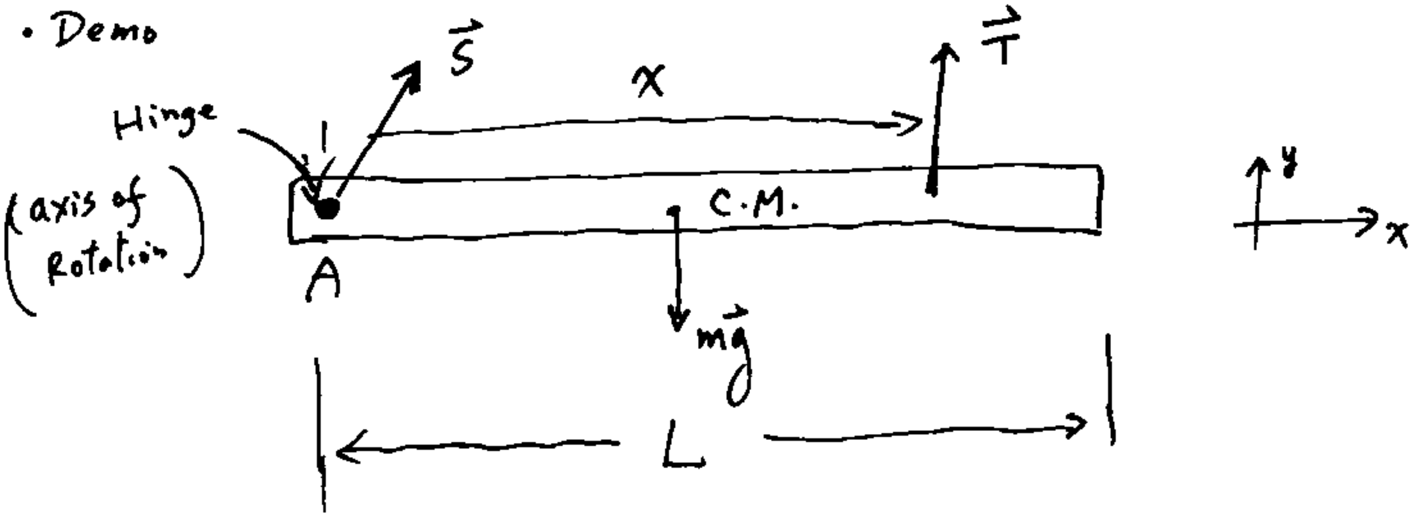
solve {① ② ③ ④} for a, T_1, T_2. α.

• static Equilibrium of a rigid body.

$$\Sigma \vec{F} = 0 \quad (\vec{a}_{c.m} = 0)$$

$$\Sigma \tau = 0 \quad (\alpha = 0)$$

• Demo



Question: How does T depend on x ?

(Note: In general, the support $\vec{S}$ can point at any direction, but in this case, it's along y, because both $m\vec{g}$ and $\vec{T}$ are vertical. $\Sigma F_x = 0 \Rightarrow S_x = 0$.)

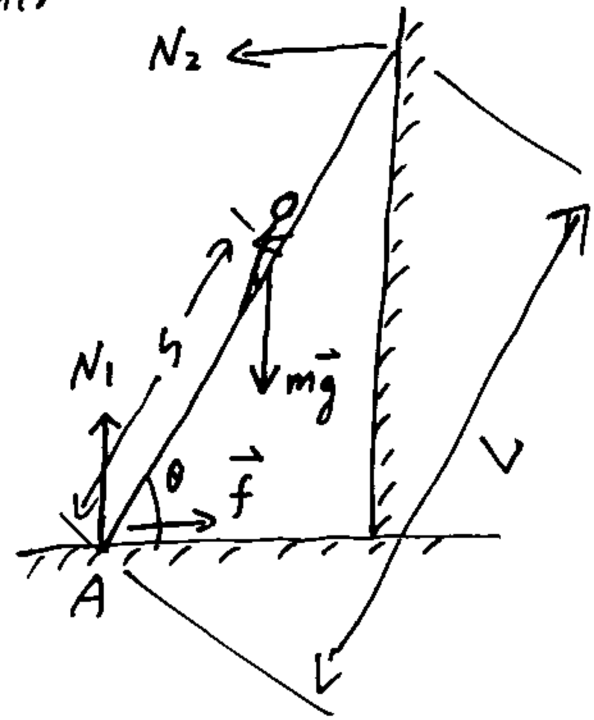
$\Sigma \vec{F}_i = 0$: $\quad S_x = 0$.

$\quad S_y + T - mg = 0$. $\quad S_y = mg - T$

$\Sigma \tau_i = 0$: choose the axis of rotation so that some unknown forces don't have τ. — Here, we choose A.

$$\left.\begin{array}{l} Tx - mg\frac{L}{2} = 0 \\ T = mg\frac{L}{2}/x \end{array}\right\} : \quad \begin{array}{ll} x = L: & T = mg/2. \\ x = \frac{L}{2}: & T = mg. \ \ (\text{C.M.}) \\ x = \frac{L}{4}: & T = 2mg. \end{array}$$

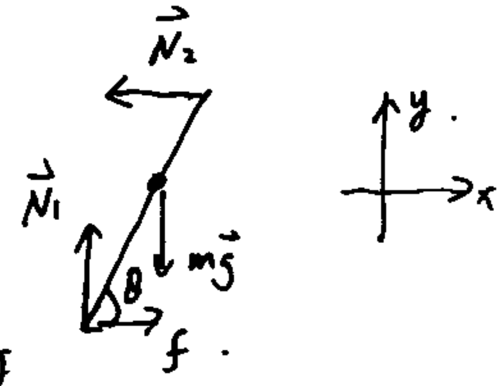
e.g: A man climbs a massless ladder against a frictionless wall. If the coefficient of friction between the ladder and the floor is μ_s, How far up the ladder can the man climb before the ladder slips?



static Equilibrium.

① $\sum \vec{F}_i = 0$

② $\sum \tau_i = 0$. (choose the axis so that many unknown forces don't have τ)



① $\begin{cases} f - N_2 = 0 \\ N_1 - mg = 0 \end{cases}$ $\quad N_1 = mg$

$N_2 = f = \mu_s N_1 = \mu_s mg$.

$f = f_{max}$ — about to slip.

② $N_2 L \sin\theta - mgs \cdot \cos\theta = 0 \Rightarrow \mu_s mg L \sin\theta - mgs \cdot \cos\theta = 0$

$$s = \frac{\mu_s L \cdot \sin\theta}{\cos\theta}$$

$$s = \mu_s L \cdot \tan\theta .$$

When the man climbs higher. Large C.W. Torque will cause the ladder to slip.