Physics 101

Lecture 19.

Friday, Feb. 20, 2004.

Topics: Angular Momentum

Conservation of Angular Momentum

Rotational Work.

Different supports.

Ref: 11-4, 5, 6, 7, 8.

- Angular Momentum
  - A quantity that describes rotational motion

(just like momentum is a quantity that describes linear motion. — also called linear momentum
$$p = mv.$$
)

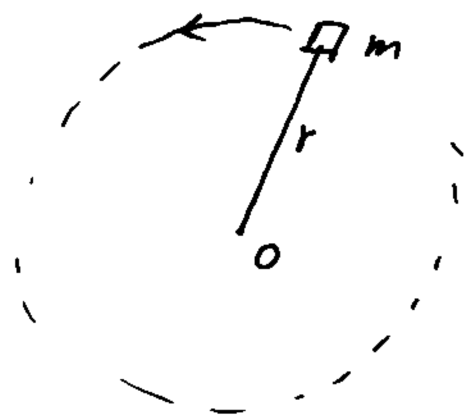
- Angular momentum of a rotating rigid body

$$L = I \cdot \omega$$

I — moment of inertia about O.

ω — angular velocity about O.

e.g. A particle at the end of a massless rod in a circular motion



$$L = I\omega$$
$$= mr^2 \cdot \omega$$
$$= mr \cdot v$$
$$= r \cdot mv$$
$$= rp.$$

$I = mr^2$

$$L = r\, p_t.$$ (angular momentum about O)

- Angular momentum of a moving particle.
  (can be moving in a straight line!)

The angular momentum of m about O:

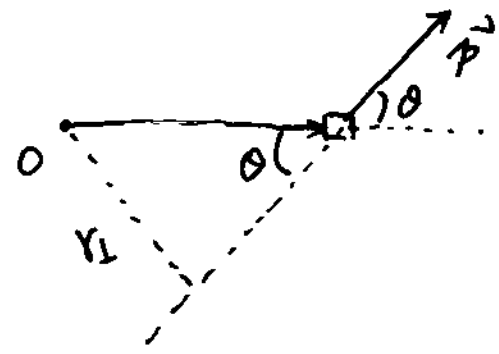
$L = r \cdot p_t$

$= r \cdot p \sin\theta$ θ — angle between $\vec{p}$ and $\vec{r}$.

$= p \cdot r \cdot \sin\theta$

$= p \cdot r_\perp$.

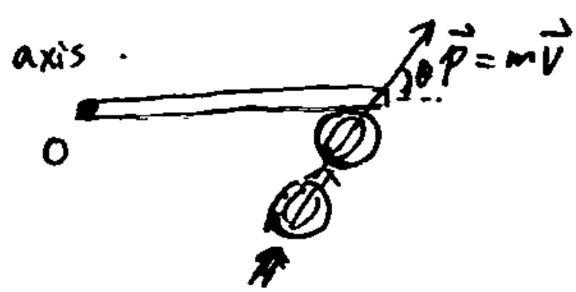
$r_\perp$ — moment arm.



- How can a particle in a linear motion have angular momentum?

  e.g: Use a basketball to open a door.

The angular momentum of the ball about the axis of the door can be transfered to the door and causes the rotation of the door



$$L = r \cdot p_t = r_\perp p = r \cdot p \cdot \sin\theta$$

- Newton's 2nd law for Rotational motion

$$\tau = I \cdot \alpha = I \cdot \frac{\Delta \omega}{\Delta t} = \frac{\Delta L}{\Delta t}.$$

$$\therefore \quad \tau = \frac{\Delta L}{\Delta t}.$$

(just like $\vec{F} = \frac{\Delta \vec{P}}{\Delta t}$)

(for a system of particles. or an extended object.
$\tau = \tau_{ext} = \frac{dL}{dt}$
$\sum \tau_{internal} \equiv 0$
L – Total angular Momentum.

- Conservation of angular momentum.

When $\tau = 0$. (No net external torque)

$$\Delta L = 0, \quad (\text{No change in } L)$$

i.e: $L = \text{const}, \quad L_i = L_f$.

e.g: Figure skater.

$\tau = 0$.

$L_f = L_i$.

$I_f \cdot \omega_f = I_i \omega_i$.

$\omega_f = \frac{I_i}{I_f} \omega_i$

Large I_i.

Small I_f.

spin faster!

If $I_f < I_i$. $\omega_f > \omega_i$ spin faster!

Demo:

I_i, ω_i.

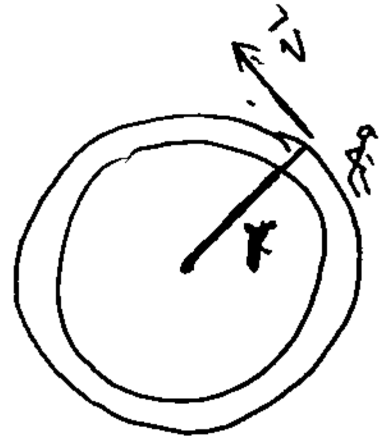
$I_f. \omega_f$.

$I_f < I_i$.

$\omega_f > \omega_i$

e.g. Run and jump onto a Merry-go-round.

(#11-5. on page 334)



Q: what's the ω of the system?

Since $\tau_{ext} = 0$.

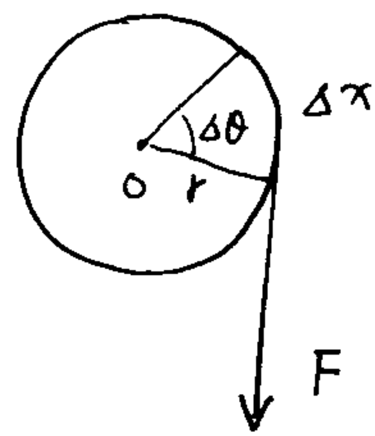
The angular momentum of the system is conserved. (The system includes the child and the Merry-go-round)

i.e:
$$L_i = L_{child} = L_f$$

$$mvr = (I + mr^2)\omega.$$

$$\omega = \frac{mvr}{I + mr^2}$$

- Rotational work.



If we pull the rope to rotate the wheel by a linear distance $\Delta x = r \cdot \Delta\theta$, the work is:

$$W = F \cdot \Delta x = F \cdot r \cdot \Delta\theta \quad (\Delta\theta \text{ — in rad})$$

i.e. $W = \tau \cdot \Delta\theta$

↑ constant torque.