Physics 101

Lecture 20.

Mon. Feb. 23, 2004

Topics: Work-Energy Relation for Rotation

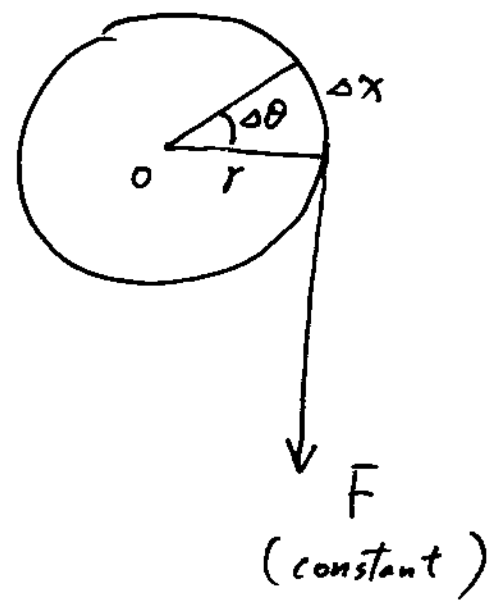
Different kinds of supports in statics

Oscillatory Motion

Ref: 11-7, 8. 13-1, 2, 3, 4

- Rotational work

$$W = F \cdot \Delta x$$
$$= F \cdot r \cdot \Delta\theta$$
$$= \tau \cdot \Delta\theta$$



$\therefore$ The Rotational work by a constant torque τ is:

$$\boxed{W = \tau \cdot \Delta\theta}$$

where $\Delta\theta$ is measured in radians.

- Work-Energy relation for rotation:

$$W = \Delta K. \quad \text{— Ture for linear and Rotational Motions !}$$

i.e. $$\tau \cdot \Delta\theta = \frac{1}{2} I \cdot \omega_f^2 - \frac{1}{2} I \omega_i^2 .$$

(τ — Net torque).

"Proof":

① . for rotation with a constant α. (i.e., const. τ).

$$\tau = \frac{\Delta L}{\Delta t} = I \cdot \frac{\Delta \omega}{\Delta t} .$$

work: $$W = \tau \cdot \Delta\theta = I \cdot \frac{\Delta\omega}{\Delta t} \cdot \Delta\theta = I \cdot \Delta\omega \cdot \frac{\Delta\theta}{\Delta t} .$$

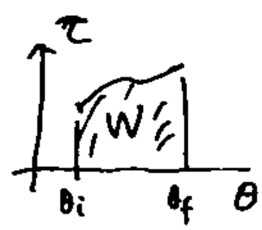
$$= I (\omega_f - \omega_i) \cdot \omega_{av} .$$

$$= I (\omega_f - \omega_i) \cdot \frac{(\omega_i + \omega_f)}{2}$$

$$= \frac{1}{2} I (\omega_f^2 - \omega_i^2)$$

$$= \Delta K .$$

② . * for Non-constant α. (τ can vary).

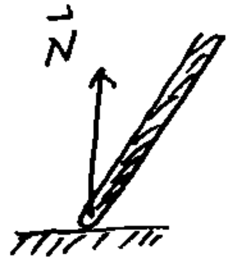


$$W = \int_{\theta_i}^{\theta_f} \tau \cdot d\theta = \int_{\theta_i}^{\theta_f} I \cdot \frac{d\omega}{dt} \cdot d\theta = \int_{\omega_i}^{\omega_f} I \cdot \frac{d\theta}{dt} \cdot d\omega$$

$$= \int_{\omega_i}^{\omega_f} I \cdot \omega \cdot d\omega = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \cdot \omega_i^2 = \Delta K_{rotation} .$$

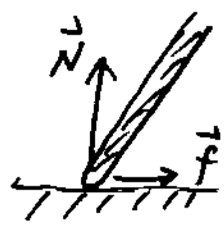
- Different kinds of supports :

1. { Rigid body. can slide No friction.



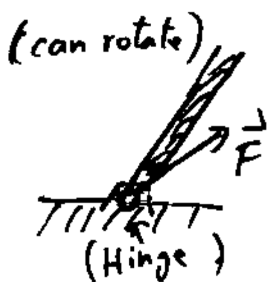
Normal force only. (Simplest)

2. { Rigid body. can slide with friction.

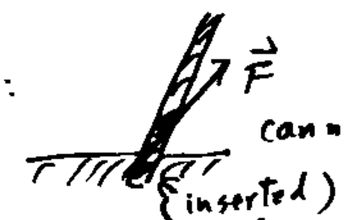


Normal force and friction. (more complicated. but. $f \leq \mu N$)

3. { Rigid body can not slide



OR :

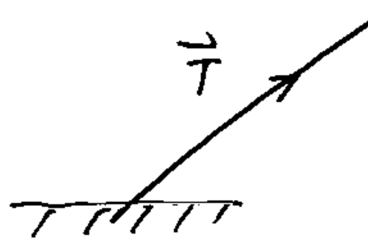


can not rotate.

$\vec{F}$ — with an unknown direction.

Usually, [redacted] $\vec{F}$ is NOT along the beam.

4. { Cable. (flexible) fixed. }



Tension $\vec{T}$ is always along the cable.

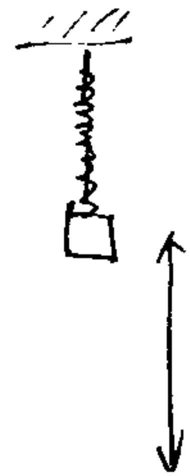
e.g. Next Monday.

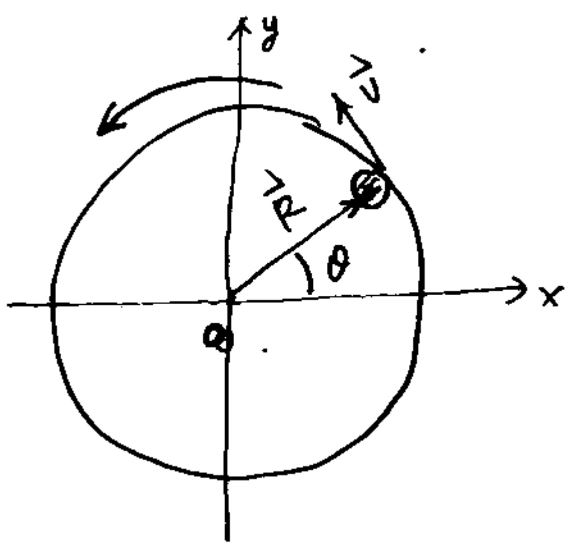
- Oscillatory Motion.

Demo: Mass on a spring – oscillates periodically

compared to: Uniform circular motion of a ping-pong ball.

Apparently, we can use the x- or y- component to describe the oscillatory motion of a vibrating block.





Reference circle.

radius $\quad R = \text{constant} \equiv A.$

angular velocity (angular frequency): $\omega = \text{constant}.$

angular position: $\theta = \omega t + \varphi.$

φ — initial angle, i.e. when $t=0$, $\theta = \varphi$.

- We can use the x-component to describe an oscillatory motion:

$$x = R\cdot\cos\theta = A\cdot\cos(\omega t+\varphi) \quad [x\text{-component of } \vec{R}]$$

$$v_x = -\omega A\cdot\sin(\omega t+\varphi) \quad [x\text{-component of } \vec{v}]$$

$$a_x = -\omega^2 A\cdot\cos(\omega t+\varphi) \quad [x\text{-component of } \vec{a}]$$

Note: for uniform circular motion,

$$\vec{a} = \vec{a}_c = -\omega^2\vec{R}. \quad (\text{centripetal acceleration})$$

- compare x with a_x:

$$\boxed{a_x = -\omega^2 x}.$$

characteristic equation of Simple harmonic motion.

Simple Harmonic Motion (SHM). — Oscillatory motion that can be described by sinusoidal functions.