Physics 101

Lecture 21.

Wed. Feb. 25, 2004

Topics: Simple Harmonic Motion (SHM)

— A vibrating block

— Simple pendulum

Ref: 13-1, 2, 3, 4, 5, 6

- comparison between uniform circular motion (U.C.M) and simple oscillatory motion (S.O.M).

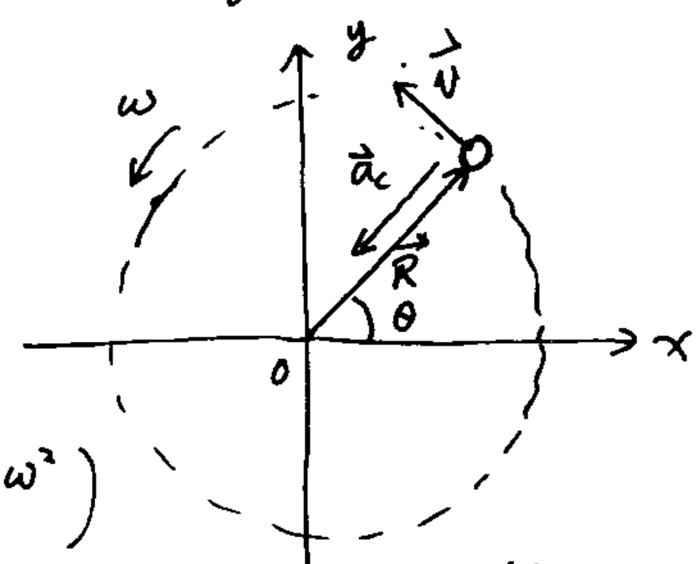
① U.C.M:

position: $\vec{R}$, radius R.

velocity: $\vec{v}$. $v = R \cdot \omega$.

acceleration: $\vec{a} = \vec{a}_c = -\omega^2 \vec{R}$

$\left(\begin{array}{l} \text{Magnitude}: \ a_c = \dfrac{v^2}{R} = R\omega^2 \\ \text{Direction}: \ -\hat{R} \end{array} \right)$



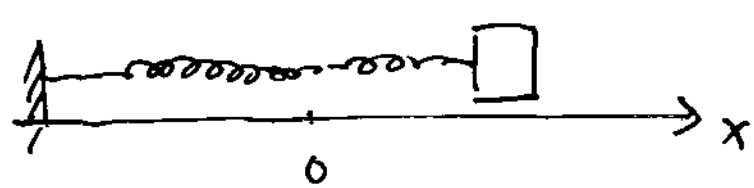
$\theta = \omega t + \varphi$.

② . S.O.M:

x-component of U.C.M.

$x = R \cdot \cos\theta = A \cdot \cos(\omega t + \varphi)$

$R = A$ A – amplitude.



- Simple Harmonic Motion (SHM).

Harmonic — sine or cosine of time.

$$\left.\begin{aligned} x &= A\cos(\omega t+\varphi) \\ v &= -\omega A\sin(\omega t+\varphi) \\ a &= -\omega^2 A\cos(\omega t+\varphi) \end{aligned}\right\} \quad x\text{-component of U.C.M.}$$

- Characteristic Equation:

$$a = -\omega^2 x$$

acceleration $\propto$ (−) displacement.

- 3 constant parameters describing the SHM: A, ω, φ.

A — Amplitude $\quad \left\{\begin{aligned} x_{max} &= A \\ v_{max} &= \omega A \\ a_{max} &= \omega^2 A. \end{aligned}\right.$

ω — angular frequency. (How quickly it oscillates)

period: $T = \frac{2\pi}{\omega}$. $\quad \omega = \frac{2\pi}{T}$.

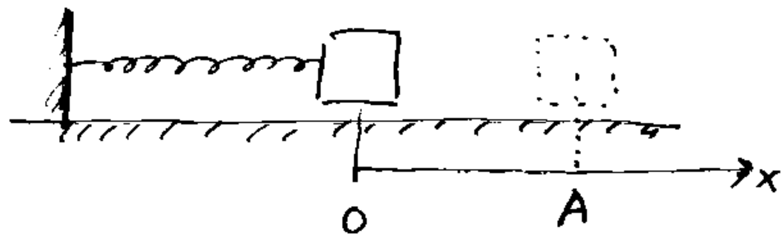
frequency: $f = \frac{1}{T} = \frac{\omega}{2\pi}$.

φ — initial phase (initial angle of the reference U.C.M).

(How and when the oscillation starts)

Text book: $\varphi = 0$. But in general $\varphi \neq 0$.

e.g.: If we pull the block to $x = A = x_{max}$, Then release it at $t=0$.



i.e., when $t=0$, $x=A$, $v=0$

Then $A = A\cdot\cos(0+\varphi) \Rightarrow \cos\varphi = 1$, $\varphi = 0$.

$$\therefore \quad x = A\cos(\omega t)$$

$$v = -\omega A\sin(\omega t)$$

$$a = -\omega^2 A\cdot\cos(\omega t).$$

(Text book case).

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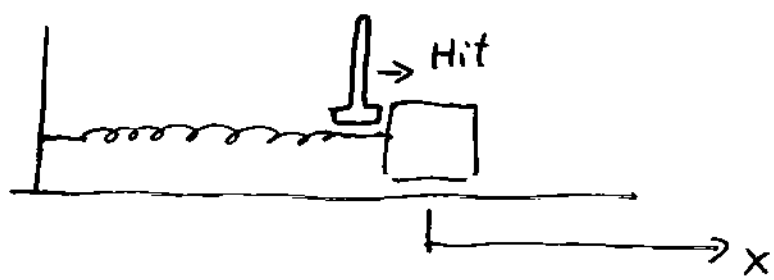
another example,

If we give the block an initial velocity $v = \omega A = v_{max}$ by hitting it to the right at $x=0$ and $t=0$.



i.e, when $t=0$, $x=0$, $v = \omega A = v_{max}$.

Then, $\left.\begin{array}{l} 0 = A\cdot\cos(0+\varphi), \\ \omega A = -\omega A\cdot\sin(0+\varphi) \end{array}\right\} \Rightarrow \begin{array}{l} \cos\varphi = 0, \ \varphi = \pm\frac{\pi}{2}. \\ \sin\varphi = -1, \ \varphi = -\frac{\pi}{2}. \end{array}$

$$\therefore \left\{\begin{array}{l} x = A\cdot\cos(\omega t - \frac{\pi}{2}) = A\cdot\sin(\omega t) \\ v = -\omega A\cdot\sin(\omega t - \frac{\pi}{2}) = \omega A\cos(\omega t) \\ a = -\omega^2 A\cdot\cos(\omega t - \frac{\pi}{2}) = -\omega^2 A\cdot\sin(\omega t). \end{array}\right] \text{Dr. Boal's case.}$$

Note: In general, φ can have any value.

$\therefore$ A and φ can be determined by initial conditions. (How and when the oscillation is started).

However, ω is determined by dynamics.
e.g. { Force, mass, etc }.

e.g: A vibrating Block, No friction.

Hooke's Law:

$$F = -kx = ma.$$

$$\therefore \quad a = -\frac{k}{m} x.$$

Compare to: $a = -\omega^2 x$ [characteristic eq. of SHM].

$$\therefore \quad \omega^2 = \frac{k}{m}. \qquad \omega = \sqrt{\frac{k}{m}}$$

k — Spring constant. (How strong the spring is)

The greater the k, the faster it oscillates.
(the stronger the restoring force)

m — mass.

The larger the m, the slower it oscillates.
(harder to accelerate)

- Mechanical Energy at any time.

$$E = U + K$$

$$= \frac{1}{2} k x^2 + \frac{1}{2} m v^2 .$$

Since there is no friction, E will be conserved.

i.e, $E = \text{const}$.

Math: $$E = \frac{1}{2} k \left[A \cdot \cos(\omega t + \varphi)\right]^2 + \frac{1}{2} m \left[-\omega A \cdot \sin(\omega t + \varphi)\right]^2$$

$$= \frac{1}{2} k A^2 \cdot \cos^2(\quad) + \frac{1}{2} m \omega^2 A^2 \cdot \sin^2(\quad)$$

$$\left[\begin{array}{l}\text{Note:} \\ \omega^2 = \dfrac{k}{m} \\ m\omega^2 = k.\end{array}\right] \Rightarrow \quad = \frac{1}{2} k A^2 \cdot \cos^2(\quad) + \frac{1}{2} k A^2 \cdot \sin^2(\quad).$$

$$= \frac{1}{2} k A^2 \left[\cos^2(\quad) + \sin^2(\quad)\right]$$

$$= \frac{1}{2} k A^2 = \text{const}.$$