Physics 101

Lecture 22.

Friday Feb. 27, 2004

Topics: SHM.

Meaning of A, ω, φ.

Mechanical Energy

parallel and series springs.

Ref: 13-1, 2, 3, 4, 5, 6

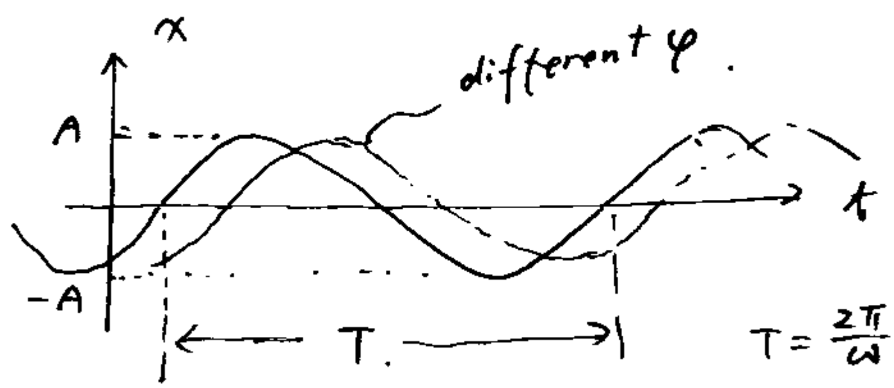
• SHM: characteristic equation: $a = -\omega^2 x$

acceleration $\propto$ (−) displacement.

$$x = A \cdot \cos(\omega t + \varphi)$$

$$v = -\omega A \sin(\omega t + \varphi)$$

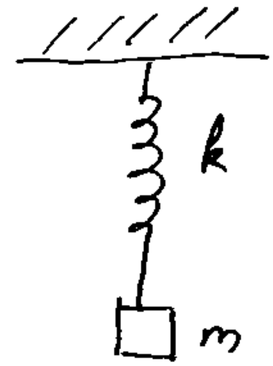
$$a = -\omega^2 A \cdot \cos(\omega t + \varphi)$$



$T = \frac{2\pi}{\omega}$. $\omega = \frac{2\pi}{T}$.

- A vibrating block.

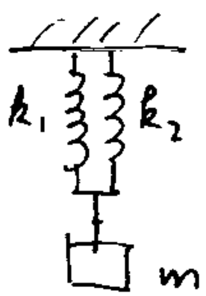
$$\omega = \sqrt{\frac{k}{m}}.$$



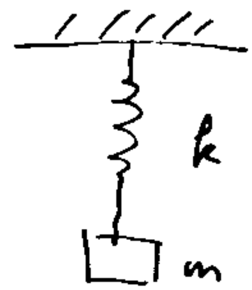
Demo:

- 2 springs.

- parallel



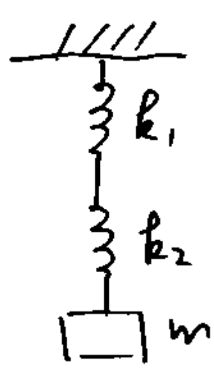
Equivalent to



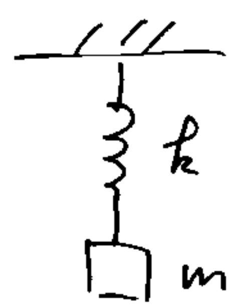
$$k = k_1 + k_2.$$

(ω — faster)

- series:



Equivalent to



$$k = \frac{k_1 k_2}{k_1 + k_2}.$$

- Mechanical Energy of a Simple harmonic Oscillator.

simple — undamped. No friction. No energy loss.

i.e., Mechanical Energy is conserved. $E = \text{Constant}$.

- Math:

At any time: $E = U + K$

$$= \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$= \frac{1}{2}k\left(A\cos(\omega t+\varphi)\right)^2 + \frac{1}{2}m\left[(-\omega A)\sin(\omega t+\varphi)\right]^2$$

$$= \frac{1}{2}kA^2\cos^2(\omega t+\varphi) + \frac{1}{2}m\omega^2A^2\sin^2(\omega t+\varphi)$$

$$= \frac{1}{2}kA^2\cos^2(\omega t+\varphi) + \frac{1}{2}kA^2\sin^2(\omega t+\varphi)$$

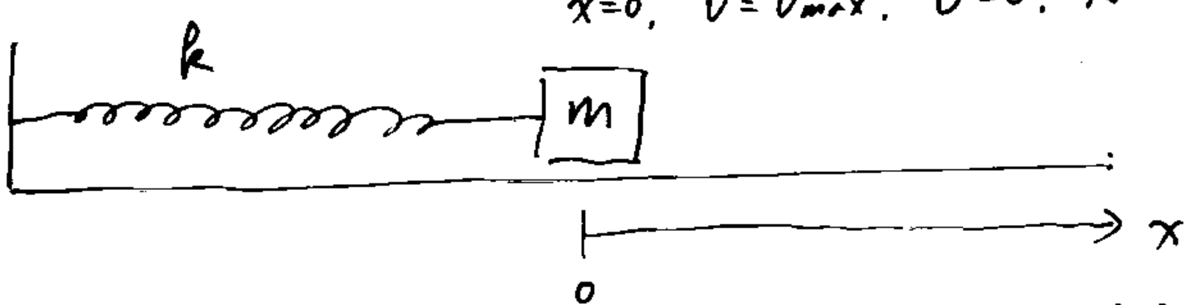
$$= \frac{1}{2}kA^2\left[\cos^2(\omega t+\varphi) + \sin^2(\omega t+\varphi)\right]$$

$$= \frac{1}{2}kA^2$$

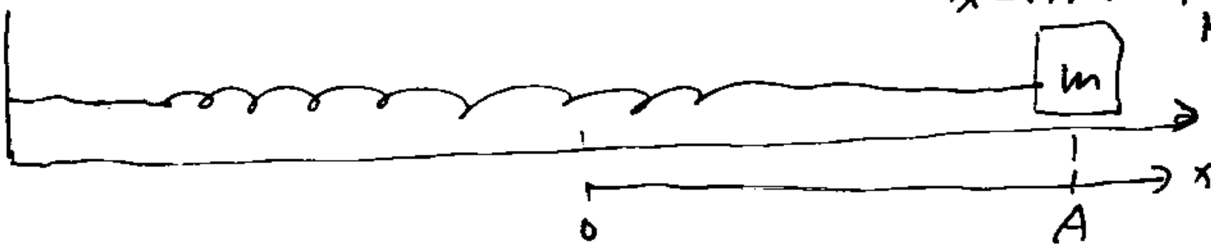
$$= \text{constant}.$$

(Note: $\omega^2 = \frac{k}{m}$, $m\omega^2 = k$)

$x=0$, $v=v_{max}$, $U=0$, $K=E$.

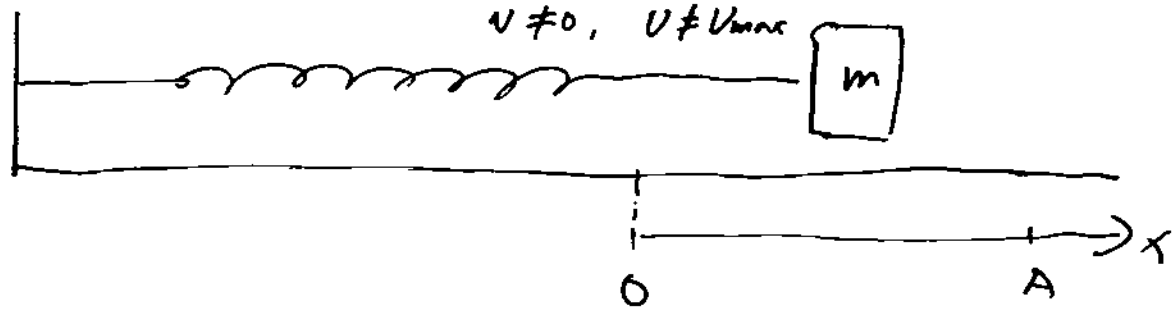


$x=A$, $v=0$, $U=E$, $K=0$



In general,

$x \neq 0, \; x \neq A. \quad U \neq 0, \; K \neq 0.$
$v \neq 0, \; v \neq v_{max}$



U and K both vary with time.

but $U + K = E$ = constant.

The Total mechanical energy is transformed between

U and K. No loss.