Physics 101

Lecture 27

Wed. March 10, 2004.

Topics : Dynamics of Rolling Motion

static equilibrium.

CAPA 7 : Due March. 23. (Tue)

CAPA 8 : Due March 30. (Tue)

- Dynamics of Rolling Motion.

$$\vec{F} = m\vec{a} \quad \text{(of C.M.)}$$

$$\tau = I\alpha \quad \text{(about C.M)}.$$

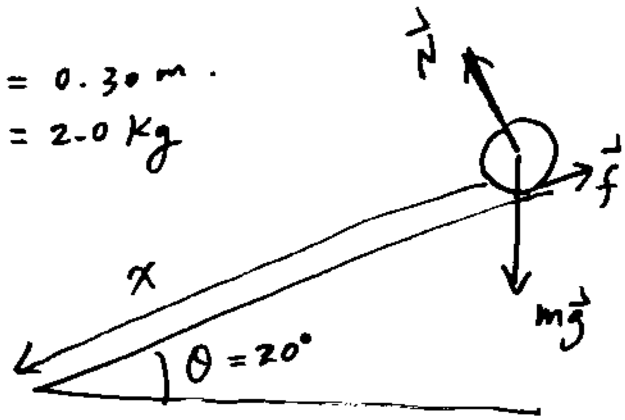
$$\boxed{\tau = I\alpha}$$

Also can be used when the axis of rotation is fixed and not necesary to be the C.M.

e.g. A solid ball rolling down an inclined plane

Initially at rest.

$r = 0.30\ m$.
$m = 2.0\ Kg$



Q1: what causes the rotation?

A torque — due to friction!

Among the 3 forces, $m\vec{g}$ and $\vec{N}$ both go through C.M.

$\therefore \quad \tau = f \cdot r$

Q2. What is the angular acceleration?

$$\begin{cases} fr = I\alpha \\ mg\sin\theta - f = ma. \\ a = \alpha r \quad \text{— rolling.} \end{cases} \qquad I = \frac{2}{5} m r^2$$

$$\begin{cases} fr = \frac{2}{5} m r^2 \alpha \quad \Longrightarrow \quad f = \frac{2}{5} m r \alpha. \\ mg\sin\theta - f = mr\alpha \end{cases}$$

$$mg\sin\theta - \frac{2}{5} m r\alpha = m r\alpha.$$

$$g\sin\theta = \frac{7}{5} r\alpha.$$

$$\alpha = \frac{5}{7r} g\sin\theta.$$

Q3: Linear acceleration of C.M.

$$a = \alpha \cdot r = \frac{5}{7} g \sin\theta$$ (independent of time) (constant acc.)

direction: along the inclined plane.

Q4. friction:

$$r f = \alpha I$$

$$f = \frac{\alpha I}{r} = \frac{\frac{5}{7r} g \cdot \sin\theta \cdot \frac{2}{5} m r^2}{r}$$

$$= \frac{2}{7} mg \cdot \sin\theta$$

check: $\vec{F} = m\vec{a}$:

x component: $mg \sin\theta - f = ma$.

$$LHS = mg \sin\theta - \frac{2}{7} mg \sin\theta = \frac{5}{7} mg \sin\theta.$$

$$RHS = m \cdot \frac{5}{7} g \sin\theta = LHS \quad \checkmark$$

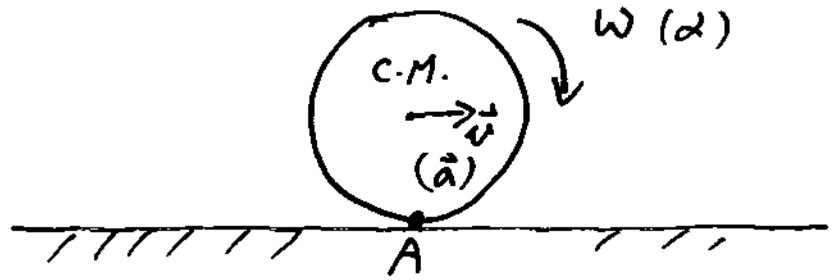
Q5. Linear velocity at x. (when $t=0$, $x=0$, $v=0$)

constant acc. $\Rightarrow$. $v = at$ $x = \frac{1}{2}at^2$, $t = \sqrt{\frac{2x}{a}}$

$$v = \sqrt{2ax}$$

you check, consistant with the result of Mechanical energy conservation.

Why $a = r\alpha$, $v = r\omega$ for rolling ?



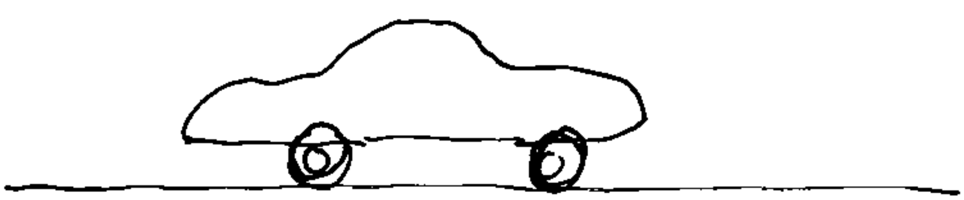
View from Ground : C.M. moves at $\vec{v}$ to the right .

View from C.M. : $\begin{cases} \text{Point A moves at } -\vec{v} \text{ to the left .} \\ \text{and } \quad v = r\omega \text{ .} \end{cases}$

$\therefore$ the speed of C.M : $v = r\omega$.

(Same as circular motion) .

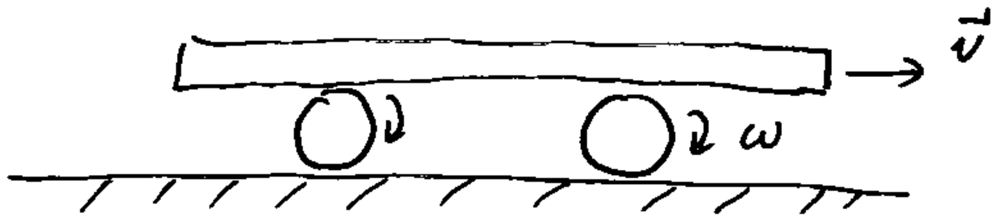
e.g:



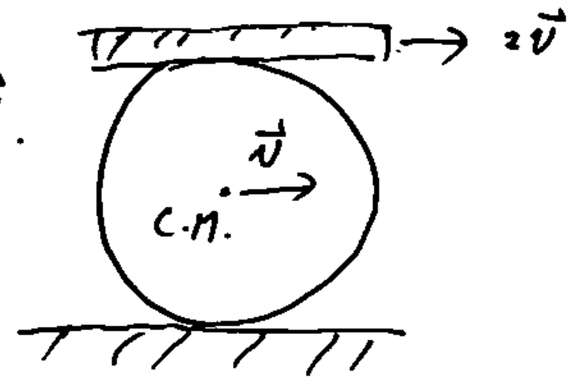
The car is moving at the same velocity of the C.M of wheels . i.e , Velocity of car $= v = r\omega$.

However, look at an unusual case:

(Q #87 on p.305)



$v = 2r\omega$. why? see p.287



View from ground:

C.M. moves to the right at $\vec{v}$.

View from C.M.

also, plank moves to the right at $\vec{v}$.

$\therefore$ The speed of plank relative to the ground:

$$\vec{v} + \vec{v} = 2\vec{v}.$$

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e.g.: 2002 Fall final Exam.

Q #6.