Physics 101

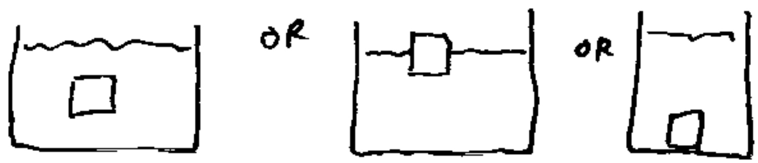
Lecture 30

Friday March 19, 2004

Topics : Streamline flow of fluids
- equation of continuity
- Bernoulli's Equation

Ref : 15-6, 7, 8

Archimede's Principle



Buoyant force = weight of fluid displaced by the object.

e.g.: Boat in a pool

If the stone is thrown into the pool, what would happen to the water level ?

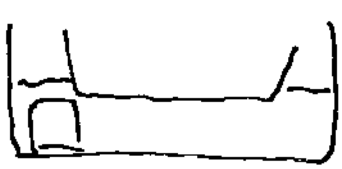
Drops ? Rises ? unchanged ? Why ?

e.g.: $V_{stone} = 1\ m^3$, $\rho_s = 2000\ kg/m^3$. $\rho_\ell = 1000\ kg/m^3$.

$m_s = 2000\ kg$.

① to float the stone, 2000 kg of water is displaced.

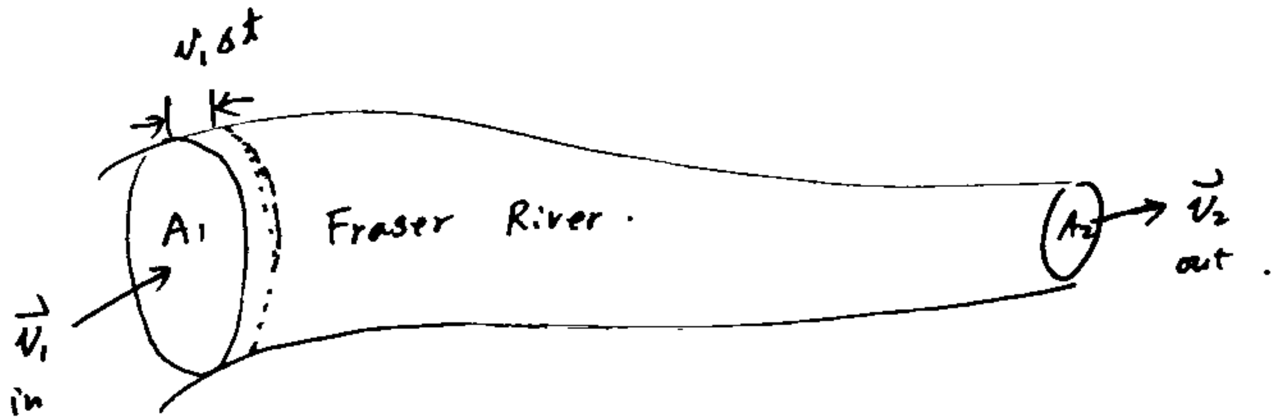
i.e, $2\ m^3$ of water is displaced.



② . when the stone is at bottom of pool. only $1\ m^3$ of water is displaced.

∴ water level drops.

• Fluid Dynamics. (Assuming No Turbulance) Laminar flow.
i.e, streamline flow.



Basic Principle: in δt,
Amount of mass in = Amount of mass out.

$$\rho_1 A_1 v_1 \delta t = \rho_2 A_2 v_2 \cdot \delta t$$

Equation of continuity

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 .$$

Same density — for incompressible fluid (Liquid).

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2} v_1 .$$

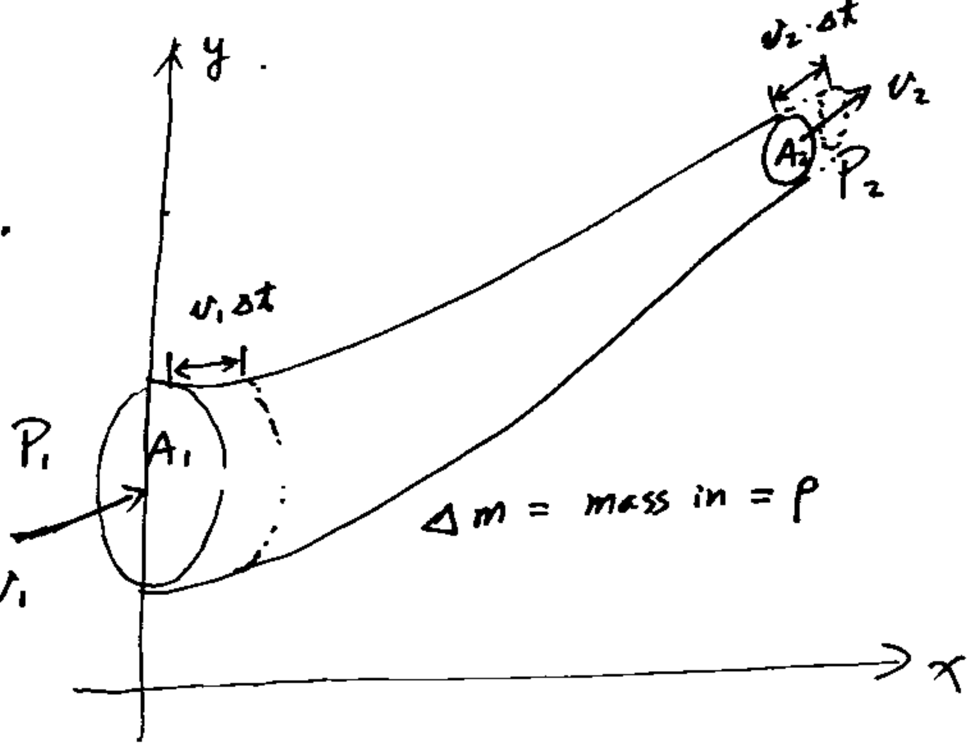
To make v_2 large at the output,
we can reduce A_2.
e.g: Nozzle of a fire hose.

Dynamics

2 Pressure can cause motion of a fluid.

Assuming same density.

In time interval Δt:



(Ignore drag).

Pressure can do work $\Longrightarrow$ Energy is increased.

$$\text{Work by Pressure} = P_1 A_1 \cdot v_1 \Delta t - P_2 v_2 A_2 \cdot \Delta t$$

$$= \frac{\Delta m}{\rho}(P_1 - P_2)$$

$$\text{gain in M.E.} = \Delta U + \Delta K$$

$$= \Delta m (y_2 - y_1) \cdot g + \frac{1}{2} \cdot \Delta m (v_2^2 - v_1^2)$$

$$\therefore \quad \frac{\Delta m}{\rho}(P_2 - P_1) = \Delta m (y_2 - y_1) g + \frac{1}{2} \Delta m (v_2^2 - v_1^2)$$

$\therefore$ Bernoulli's Equation :

$$\boxed{\begin{aligned} &P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2 \\ &\text{OR:} \quad P + \rho g y + \tfrac{1}{2}\rho v^2 = \text{constant} . \end{aligned}}$$

- Special Case

When $y_1 = y_2 = \text{const}$.

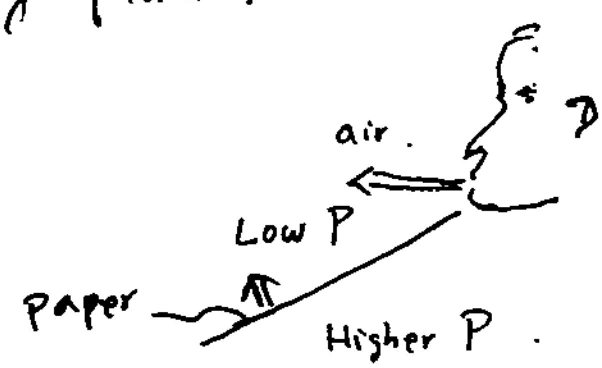
B. Eq : $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$.

OR : $P + \frac{1}{2}\rho v^2 = \text{const}$.

There is a tradeoff between P and v^2 .

e.g : Moving fluids have a lower pressure than stationary fluid .

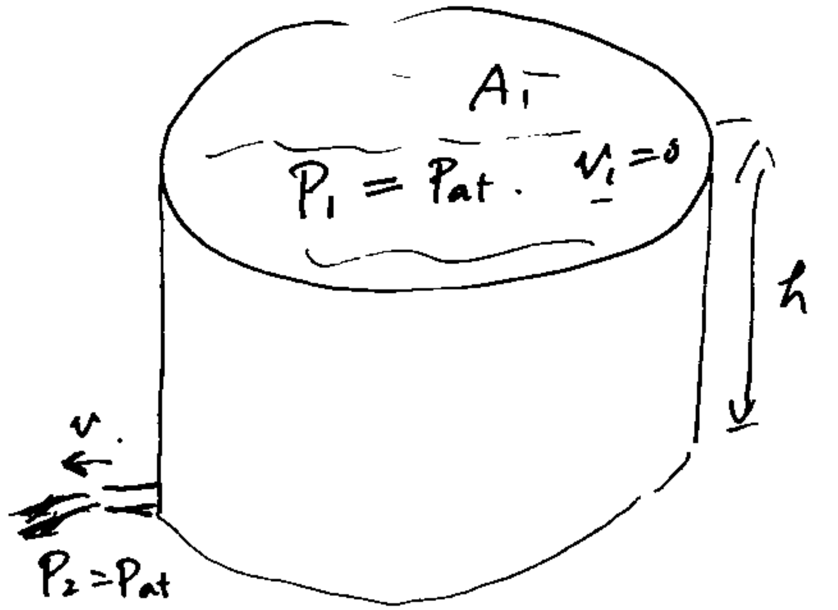
Demo :



e.g: Torricelli's Law.

Large container with a small hole near the bottom.

What's the speed of the ~~flow~~ fluid when it flows out of the small hole?



B. Eq: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$.

$\because$ $P_1 = P_2 = P_{at}$,

$v_1 = 0$. since A_1 is very large.

$\therefore$ $\rho g y_1 = \frac{1}{2}\rho v_2^2 + \rho g y_2$.

$\therefore$ $v_2^2 = 2g(y_1 - y_2) = 2gh$.

$v_2 = \sqrt{2gh}$.

Conservation of Mechanical Energy. (just like a free-falling object).