Physics 101

Lecture 31

Monday, March 22, 2004

Topics : Bernoulli's Equation

Torricelli's Law.

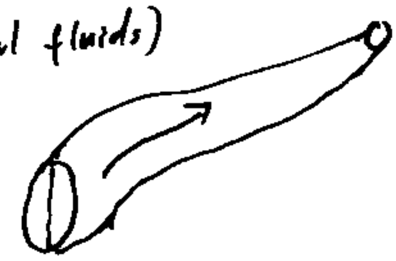
Viscosity.

Ref : 15-6, 7, 8, 9

- Fluid Dynamics
  - Pressure can cause motion of fluid.

- Bernoulli's Equation : $P + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$.
  - Assuming No drag (ideal fluids)
  - Relates P, v, y

    P ↑ Pressure, v ↑ velocity, y height.



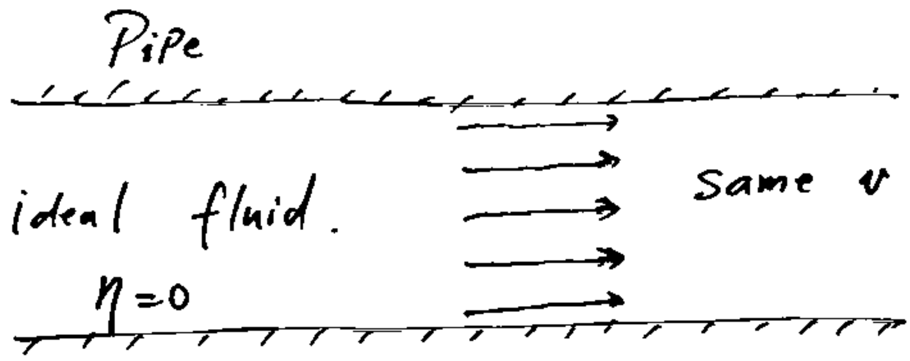
  - Derivation : see lecture Note #30, page 30-3.

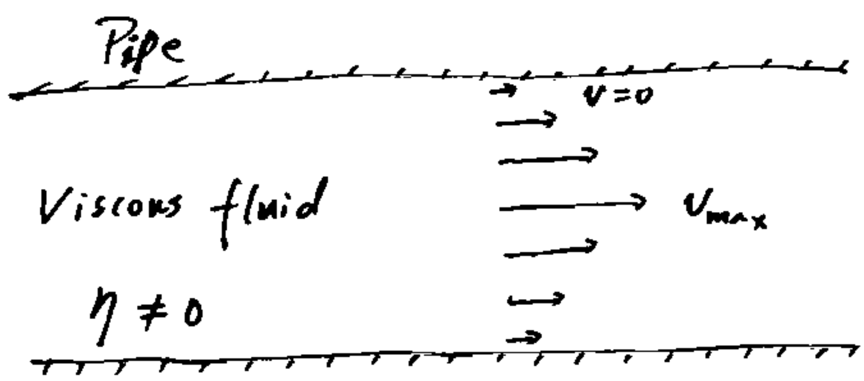
- Torricelli's Law.
  - see lecture #30-5.

- Viscosity (η) — Drag.

Up till now, we have been dealing with ideal fluids — No viscosity (i.e., No friction, No drag).

No interaction between the molecules in the fluid.



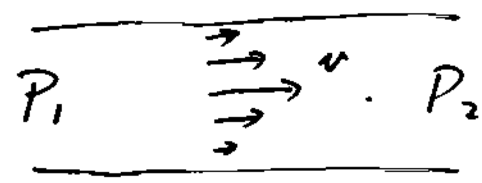


$\eta \neq 0$ because of interactions $\begin{cases} \text{between pipe and fluid} \\ \text{among molecules in the fluid.} \end{cases}$

- Pressure difference can cause the flow of viscous fluid

$$P_1 - P_2 \propto v.$$

v — average speed.



More specifically:

$$P_1 - P_2 = 8\pi\eta \frac{vL}{A}.$$

L — length of pipe.

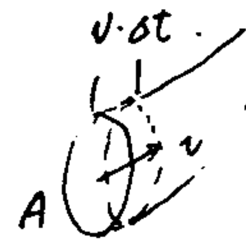
A — cross section area.

η — viscosity in $[N \cdot s / m^2]$.

another unit: poise $= 0.1\ N \cdot s / m^2$.

- Volume flow rate.

(Volume per unit time).



$$Q = \frac{\Delta V}{\Delta t} = \frac{A v \cdot \Delta t}{\Delta t} = A v .$$

$$\therefore \quad Q = \frac{(P_1 - P_2)\, A^2}{8 \pi \eta L}$$

for a cylindrical tube :

$$Q = \frac{(P_1 - P_2)\, \pi r^4}{8 \eta L} . \qquad (\text{Poiseuille's equation}).$$

$$\therefore \quad Q \propto r^4 .$$

In an artery, reducing r (due to fat, for example) will cause increasing blood pressure dramatically to maintain certain blood flow rate Q.

e.g.: last semester's final exam. #15.