- Typical Mistakes.
  - — jump to equations without setting up a coordinate system.
  - — Always assume constant acceleration. or even constant velocity.

3. Dynamics.

- Free-Body diagram.
  - — Forces : $\begin{cases} \text{contact forces} \\ \text{field forces (gravity)} \end{cases}$.
  - — Always set up your coordinate system.
- Equations of Motion — instantaneous relationship.
  - Linear (can be 3-D): $\vec{F} = m\vec{a}$.
  - Rotation of a rigid body: $\tau = I\alpha$

  Note: There is No instantaneous relation between the net force $\vec{F}$ and the velocity!

- Typical Mistakes $\begin{cases} \text{forces missing.} \\ \text{wrong torques } (\sin\theta \text{ vs } \cos\theta). \\ \text{forget Kinematics relations} \end{cases}$
  e.g.: $a = r\cdot\alpha$.

4. statics of a rigid body.

- static equilibrium: $\vec{F}_{net} = 0$. $(\because \vec{a} = 0)$

  $\tau_{net} = 0$. $(\because \alpha = 0)$.

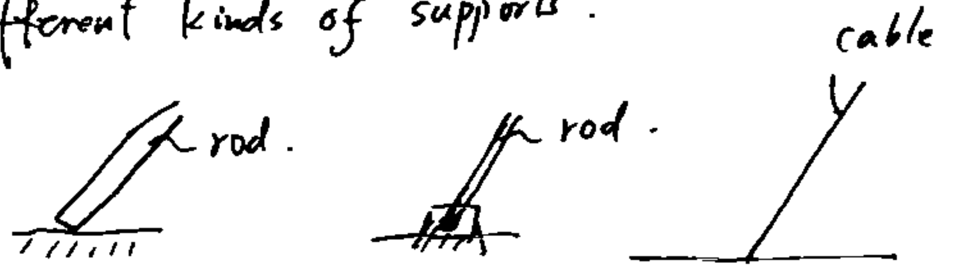
- Axis of rotation for calculating $\tau_{net} = \Sigma \tau_i$.

  — arbitrary. since there is no actual rotation.

  · Then, choose a ~~conenient~~ convenient one.

  i.e. unknown forces have zero torques.

- Different kinds of supports.



- Typical mistakes.

  — Can't recognize it's a statics problem.

  — wrong assumption about the direction of an unknown force.

  — Mistakes in torque calculations.

36-9.
37-3.

5. Quantities of Motions.

- Momentum $\vec{p} = m\vec{v}$.

$$\Delta\vec{p} = \vec{F}\cdot\Delta t = \overrightarrow{\text{Impulse}}. \quad (\text{Impulse-Momentum Thm})$$

Time-cumulative effect of the net force.
$\Rightarrow$ change in momentum.

- Kinetic energy. $K = \frac{1}{2}mv^2$.

$$\Delta K = \vec{F}\cdot\Delta\vec{x} = \text{Work}. \quad (\text{work-energy Thm})$$

Space-cumulative effect of the net force
$\Rightarrow$ change in kinetic energy.

Note: for a many-body system.

$$\Delta K_{\text{Total}} = \text{Work}_{\text{Total}} = \sum \vec{F_i}\cdot\Delta\vec{x}_i$$

Internal forces contribute to $\text{Work}_{\text{Total}}$!
But internal forces don't contribute to $\text{Impulse}_{(\text{Total})}$

- Angular Momentum of a rigid body. $L = I\omega$.

$$\Delta L = \tau\cdot\Delta t.$$

- Rotational Kinetic Energy. $K_R = \frac{1}{2}I\omega^2$.

$$\Delta K_R = \tau\cdot\Delta\theta.$$

36-40
37-4.

- Typical mistakes
  - Can't recognize the correspondence
    - time ~ momentum: $\Delta t \cdot \vec{F} = \Delta \vec{p}$
    - distance ~ Kinetic Energy: $\Delta \vec{x} \cdot \vec{F} = \Delta K$
  - Can't use the idea of $\Delta(\ ) = (\ )_f - (\ )_i$.

    i.e., we are only interested in the final and initial states. i.e, the states before and after interaction.

6. Conservative and non-conservative forces.

- Differences:
  - Whether or not work depends on path.
  - whether or not work can be stored as potential energy.

- Potential energy. U:

$$\Delta U = -W_c. \quad \text{OR.} \quad -\Delta U = W_c.$$

~~The conservative work~~

The work done by a conservative force (W_c) equals the decrease in Potential Energy. ($-\Delta U$)

7. Generalized work-energy theorm

$$W_{nc} = \Delta E \qquad \left(\because \; W_{Total} = \Delta K . \right.$$
$$= \Delta K + \Delta U \qquad W_c + W_{nc} = \Delta K$$
$$W_{nc} = \Delta K - W_c$$
$$= \Delta K + \Delta U$$

Non-conservative work done on the system equals the increase in Mechanic energy.

8. Conservation Laws:

— when $W_{nc} = 0$, $\Delta E = 0$
i.e., when there is no non-conservative work, the mechanical energy is conserved.

— when $\vec{F}_{net} = 0$, $\Delta \vec{P} = 0$
i.e. when the net external force is zero, The momentum is conserved.

— when $\tau = 0$, $\Delta L = 0$.
i.e. when the total external torque is zero. The angular momentum is conserved.

9. Collisions.

— when $\vec{F}_{net}$ is zero or can be neglected,
Then, $\vec{P} = \text{const}$, (Total momentum is conserved)

— when the collision is elastic.
Then, total kinetic energy is conserved.

— when the collision is completely inelastic,
Then the two objects have the same final velocity. ~~(They got stuck~~ (They stick to each other).

· Typical Mistakes:
— Don't know the conditions for conservation laws.
— unable to solve systems of algebraic equations.

10. Centre of mass. $\vec{x}_c = \dfrac{\sum m_i \vec{x}_i}{M}$, $\vec{v}_c = \dfrac{\sum m_i \vec{v}_i}{M}$.

i.e. $M\vec{v}_c = \sum m_i \vec{v}_i$

$\vec{P}_c = \vec{P}_{Total}$.

$\vec{a}_c = \dfrac{\sum m_i \vec{a}_i}{M}$.

11. Friction and Drag.

– Friction: static $f \leq \mu_s \cdot N$.

$f_{max} = \mu_s N$.

Kinetic $f = \mu_k \cdot N$.

– Drag: $f \propto v$ — slow. (without turbulance)

$f \propto v^2$ — fast (with turbulance).

– Terminal velocity of a falling object. $\Big\}\ f(v_T) = mg$.

$v = \text{const}$. eventually

12. Centripetal force. — Required by Newton's 2nd law of Motion.

$\vec{F}_c = m\vec{a}_c$

$F_c = m \cdot \frac{v^2}{R} = mR\omega^2$.

13. Rotational Motion

- Kinematics

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}.$$

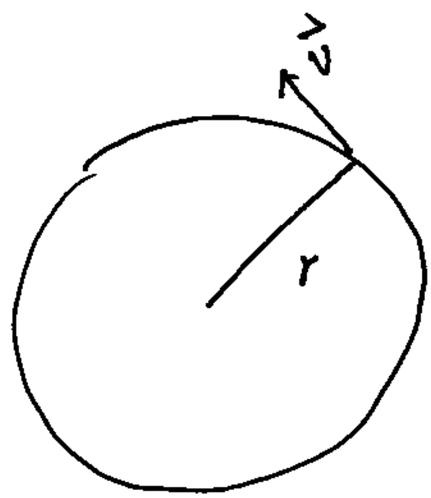
$$s = r \cdot \Delta\theta \quad (\theta - \text{radians})$$

$$v = r \cdot \omega$$

$$a_c = \frac{v^2}{r} = r \cdot \omega^2$$

$$a_t = r \cdot \alpha$$

$$\vec{a} = \vec{a}_c + \vec{a}_t \quad (\text{vector!})$$



$$\theta(t) \xrightarrow{\text{slope}} \omega(t) \xrightarrow{\text{slope}} \alpha(t)$$

$$\theta(t) \xleftarrow[\text{area}]{} \omega(t) \xleftarrow[\text{area}]{} \alpha(t)$$

- Dynamics.

$$\tau = \frac{\Delta L}{\Delta t} = I \cdot \alpha .$$

τ — not torque

I — moment of inertia

α — angular acceleration

L — angular momentum.

Many-body systems. $\Sigma \tau_{in} = 0$.

$$\tau_{ext} = \frac{\Delta L}{\Delta t} .$$

- conservation of angular momentum.

when $\tau_{ext} = 0$.

$\Delta L = 0$, $L = \text{const}$. $L_f = L_i$.

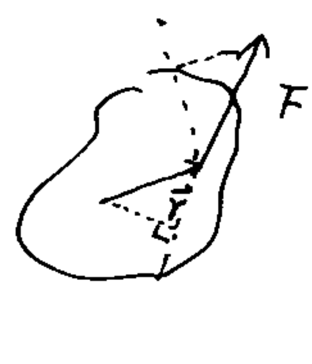
14. statics.

when $\alpha = 0$, $\vec{a} = 0$. ($\vec{a}_{c.m} = 0$)

Then $\Sigma \tau = 0$, $\Sigma \vec{F}$[redacted] $= 0$.

choose an "axis of rotation" so that many unknown forces have no contribution to τ.

- The torque of a force.



$$\tau = F \cdot r \cdot \sin\theta = \begin{cases} F \cdot r_\perp . \\ \text{OR.} \\ F_t \cdot r . \end{cases}$$

15. Simple Harmonic Motion.

characteristic equation: $a = -\omega^2 x$.

$(\alpha = -\omega^2 \theta)$.

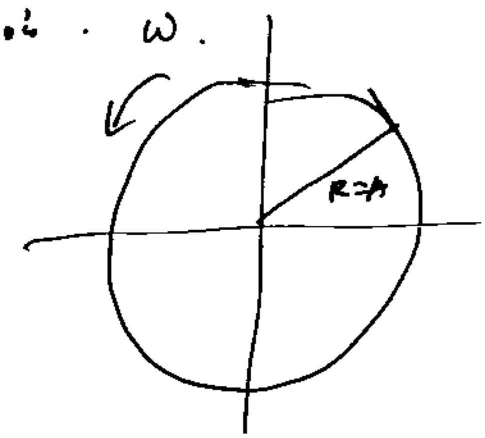
displacement: $x = A \cdot \cos(\omega t + \varphi)$

velocity: $v = -\omega A \cdot \sin(\omega t + \varphi)$

acceleration: $a = -\omega^2 A \cdot \cos(\omega t + \varphi)$

- Reference circle:

SHM can be described by the x-component of a uniform circular motion. ω.



$x_{max} = A$.

$v_{max} = \omega A$

$a_{max} = \omega^2 A$.

- A, ω, φ.

A — amplitude.

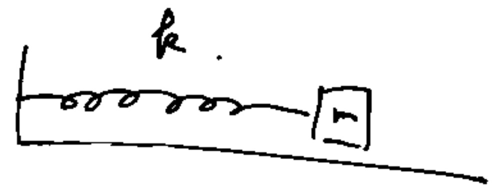
ω — angular frequency. $T = \frac{2\pi}{\omega}$, $f = \frac{\omega}{2\pi}$.

φ — initial phase.

~~A, φ, — can be determin~~

- vibrating block.

$$F = ma.$$



$$-kx = ma.$$

$$a = -\frac{k}{m}x. \qquad (a = -\omega^2 x).$$

$$\omega = \sqrt{\frac{k}{m}}.$$

- Simple pendulum.

$$\omega = \sqrt{\frac{g}{L}}$$

16. Waves, — propagation of SHM.

$$y = A\cdot\cos(kx - \omega t + \varphi).$$

$$= A\cdot\cos\left[k(x - vt) + \varphi\right]. \qquad k\cdot v = \omega.$$

$$k = \frac{2\pi}{\lambda}. \qquad \omega = \frac{2\pi}{T}. \qquad v \text{ — speed of wave.}$$

$$v = \frac{\lambda}{T} = \lambda f.$$

- Intensity and Intensity level of sound.

intensity.

$$I = \frac{P}{A}.$$

I — intensity $\quad P = E/t.$

P — power

A — area.

intensity level:

$$\beta = 10 \log \left(\frac{I}{I_0}\right).$$

in dB.

$I_0 = 1 \times 10^{-12}\ W/m^2.$

(Threshold).

- Doppler effect.

$$f' = f \left(\frac{1 \pm \frac{u_o}{v}}{1 \mp \frac{u_s}{v}} \right)$$

Sign: approaching, f' shifts to higher freq.

receding, f' shifts to lower.

- Superposition principle.

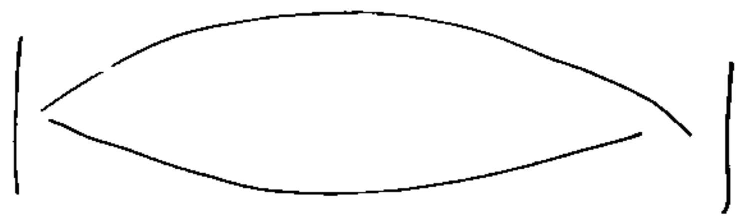
$$y = y_1 + y_2.$$

- Beats.

$$\lambda_B = \text{common multiple of } \lambda_1 \text{ and } \lambda_2.$$

$$f_B = |f_1 - f_2|$$

- standing waves.



$$L = \frac{\lambda}{2}(n).$$

$$\lambda = \frac{2L}{n}.$$

$$f = \frac{v}{2L}\cdot(n).$$

$n=1$ — fundamental

$n=2$ — 2nd harmonic.

$n=3$ — 3rd harmonic

... ...

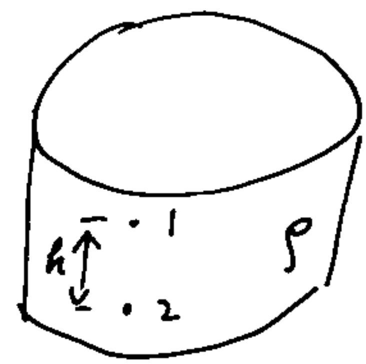
17. Fluid statics.

— Apply Newton's 2nd Law, but $\vec{a} = 0$

- Pressure

$$P = F/A.$$

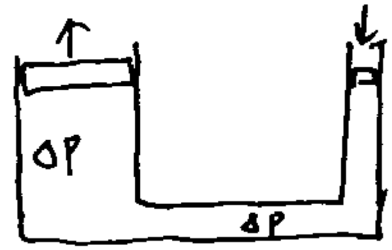
$$P_2 = P_1 + \rho g h$$



- Pascal's principle.

(Hydraulic lift)

same ΔP everywhere



- Buoyancy.

$$B = \rho_\ell \cdot g \cdot V$$

V — displaced volume.

18. Fluid dynamics

- Equation of continuity: $\rho A v = \text{const}.$ (mass in = mass out.)

  for incompressible fluid (e.g. Liquid).

  $$Av = \text{const}.$$

  OR: $A_1 v_1 = A_2 v_2$.

- Bernoulli's equation. (Work of Pressure $= \Delta E$)
  (Ignore viscosity)

  $$P + \rho g y + \frac{1}{2}\rho v^2 = \text{const}.$$

  when $y = \text{const}$.

  $$P + \frac{1}{2}\rho v^2 = \text{const}.$$

- Viscous flow:

  Volume flow rate: $$Q = \frac{(P_1 - P_2)A^2}{8\pi\eta L}.$$

  for a cylindrical tube:

  $$Q = \frac{(P_1 - P_2)\pi r^4}{8\pi\eta}.$$

19. Temperature and Heat.

- Temperature scales.

  Best : Kelvin.

- Thermal expansion

  $\cancel{\beta = 3\alpha}$.

  Linear: $\Delta L = \alpha \cdot L_0 \cdot \Delta T$.

  area: $\Delta A = 2\alpha \cdot A_0 \cdot \Delta T$.

  Volume: $\Delta V = 3\alpha \; V_0 \cdot \Delta T$

  OR: $\Delta V = \beta \cdot V_0 \Delta T$.

- Conduction.

  Heat flow: $Q = k \cdot A \cdot \left(\frac{\Delta T}{L}\right) t$.

  flow rate: $H = k A \cdot \left(\frac{\Delta T}{L}\right)$.

- Radiation:

  power: $P = e \sigma A T^4$.

  $e = 1$ — black body. (perfect emitter)

20. Heat Capacity and Specific heat.

$$C = \frac{Q}{\Delta T} \quad \text{— for an object.}$$

$$c = \frac{C}{m} = \frac{Q}{m \cdot \Delta T} \quad \text{— for a substance.}$$

21. Ideal gases and Kinetic Theory

$$\frac{PV}{T} = Nk = nR$$

$$\frac{3}{2} kT = \left(\frac{1}{2} m v^2\right)_{ave}.$$

$$v_{rms} = \sqrt{\frac{3kT}{m}} \qquad v_{rms} \equiv \sqrt{(v^2)_{ave}}.$$

suggestions: (for practice)

① CAPA
② old Tests.
③ Lecture Notes
④ Textbook examples.