Physics 102

Lecture 5. Friday Sept. 17, 2004

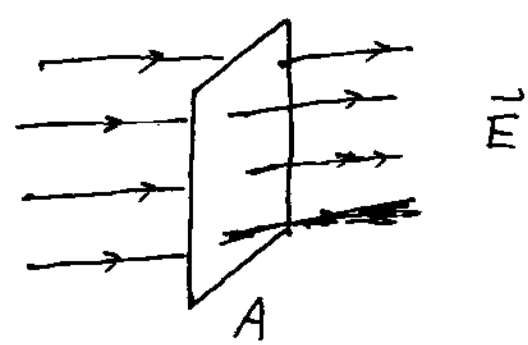
- Flux Φ

— A useful concept for calculating $\vec{E}$ using Gauss's Law.

— What is flux?

Roughly speaking, electric flux $\propto$ # of $\vec{E}$-lines going through a given area.

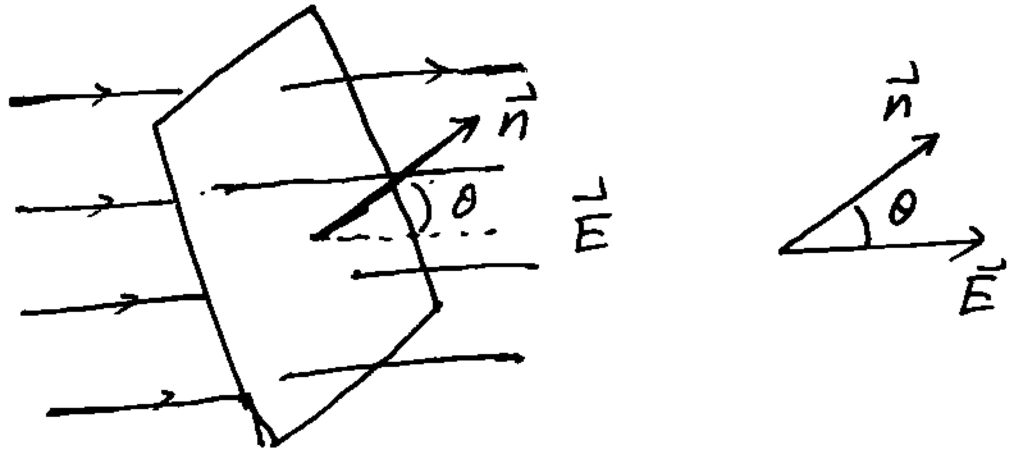
① A uniform electric field passing through an area that is perpendicular to the field.



Then, $\Phi = E A$

unit of electric flux: $N \cdot m^2/C$

② . The uniform field is not perpendicular to the area .



Then , $\Phi = EA\cos\theta = \vec{E}\cdot\vec{n}\cdot A = \vec{E}\cdot\vec{A}$.

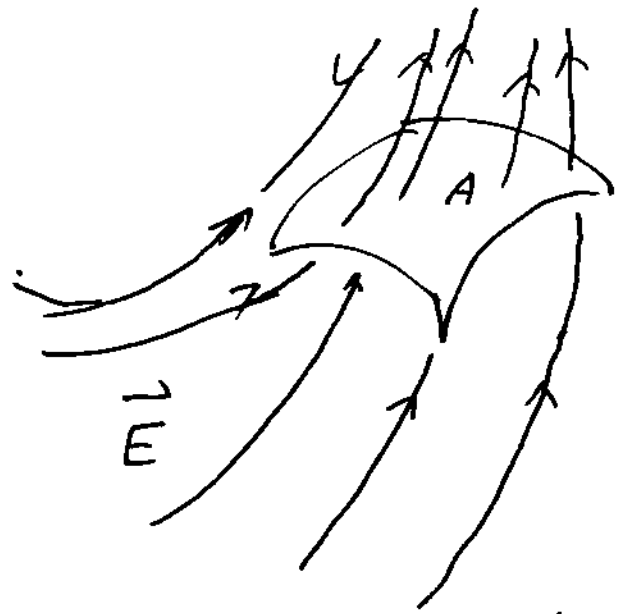
(direction of $\vec{A}$ = direction of normal)

which normal ? Left or right ?

— it doesn't matter .

But, for an enclosed area, $\vec{n}$ — outward

③. In general, $\vec{E}$ is not uniform, A is not a plane area.



Divide A into many small ΔA_i, each one can be considered as a plane area with a well-defined normal $\cdot \vec{n}_i$, $\overrightarrow{\Delta A_i} = (\Delta A) \cdot \vec{n}_i$.

Within each small ΔA_i, $\vec{E}_i$ can be considered as uniform.

The flux through ΔA_i : $\Delta \Phi_i = \vec{E}_i \cdot \overrightarrow{\Delta A_i}$.

Then, the total flux through A :

$$\Phi = \sum \Delta \Phi_i = \sum \vec{E}_i \cdot \overrightarrow{\Delta A_i} = \sum E_i \cdot \Delta A_i \cdot \cos\theta_i .$$

$$= \int \vec{E} \cdot \overrightarrow{dA} \left(= \iint E \cdot \cos\theta \cdot dS \right)$$

↑ Key idea.

arbitrary

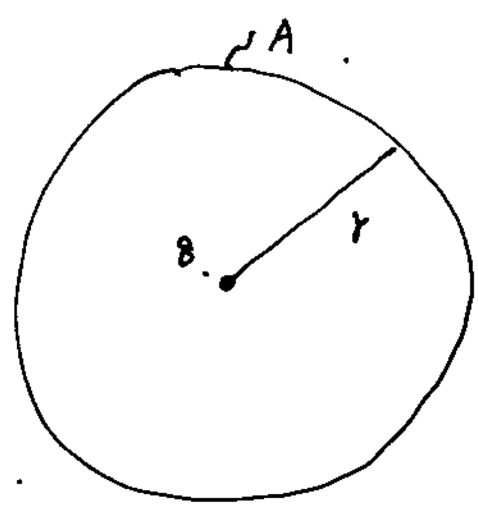
- Gauss's law.

If a charge q is enclosed by an arbitrary surface, the electric flux through the surface is :

$$\boxed{\Phi = \frac{q}{\varepsilon_0}} \quad (N \cdot m^2/C)$$

ε_0 — permittivity of free space.

e.g: The electric flux of a spherical surface due to a ~~ch~~ point charge q at the centre.



$$\Phi = \sum \vec{E}_i \cdot \vec{\Delta A}_i$$

On the surface :

$$\vec{E}_i = \underbrace{\frac{kq}{r^2}}_{\text{same everywhere on sphere}} (\hat{r})$$

$$\vec{\Delta A}_i = \Delta A_i \cdot \hat{r}$$

$$\therefore \Phi = \frac{kq}{r^2} \sum \Delta A_i (\hat{r} \cdot \hat{r}) = \frac{kq}{r^2} \sum \Delta A_i = \frac{kq}{r^2} \cdot 4\pi r^2 .$$

$\Phi = 4\pi k q$. Gauss's law: $4\pi k q = \dfrac{q}{\varepsilon_0}$

$$\therefore \varepsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \ C^2/N \cdot m^2$$