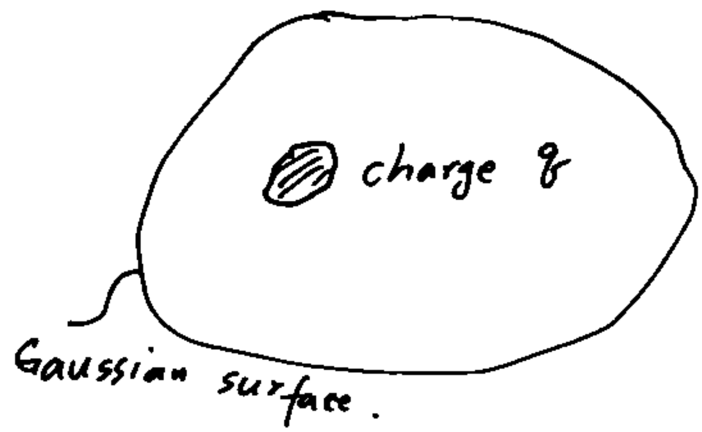
Physics 102

Lecture 6 Monday Sept. 20, 2004.

- Gauss's Law.

$$\Phi = \frac{q}{\varepsilon_0} . \qquad \Phi = \Sigma \vec{E} \cdot \vec{\Delta A}$$




- Application of Gauss's Law:

$\underset{\uparrow \text{ by symmetry.}}{\text{choose Gaussian surface}}$ to simplify $\Sigma \vec{E} \cdot \vec{\Delta A}$.

- Applications of Gauss's Law

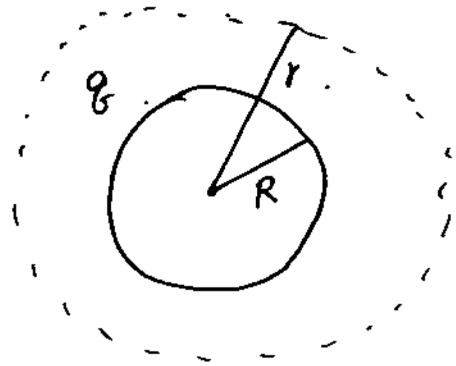
e.g.: A solid sphere has a radius $R$ and charge $q$.

The charge is uniformly distributed in the sphere. (NOT Metal)

Find the electric field.

charge density: $\rho = \frac{q}{V}$

$= \frac{3q}{4\pi R^3}$




$V = \frac{4}{3}\pi R^3$

① spherical Gaussian surface => sphere.

$r > R$

by symmetry:

$$\Phi = \sum \vec{E}_i \cdot \Delta \vec{A} \quad \left( = \frac{q}{\varepsilon_0} \right)$$

$$= \sum E \, \Delta A_i$$

$$= E \sum \Delta A_i$$

$$= E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$\vec{E}_i = E_i \hat{r}$

$= E(r)\, \hat{r}$

depends on $r$ only.

(surface area of a sphere: $A = 4\pi r^2$)

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}, \quad \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^3} \cdot \vec{r}$$

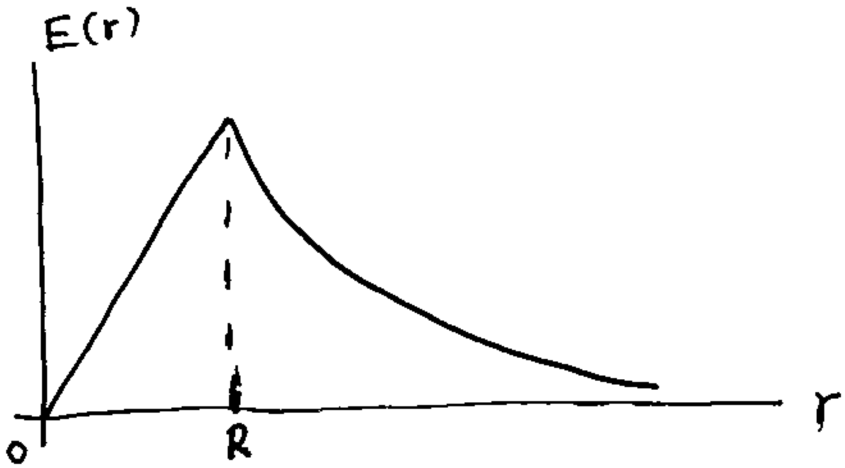
$\left( \hat{r} = \frac{\vec{r}}{r} \right)$

② $r < R$.

$$\Phi = E \cdot 4\pi r^2 \qquad \text{but. charge} = \rho \cdot V_r = \rho \cdot \frac{4}{3}\pi r^3$$

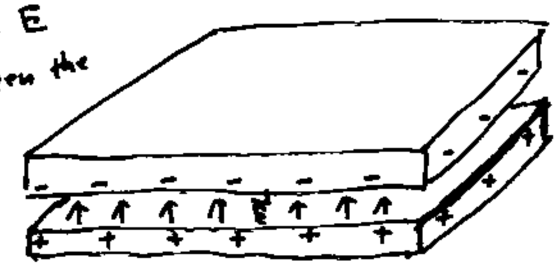
$$E\, 4\pi r^2 = \frac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0} = \frac{3q \; \frac{4}{3}\pi r^3}{4\pi R^3 \varepsilon_0}$$

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0} \frac{q r}{R^3}; \quad \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^3} \vec{r}$$



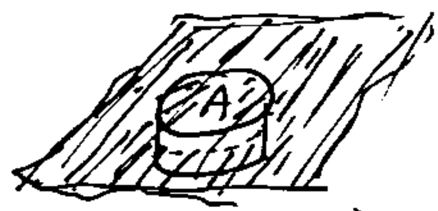

e.g. 19-3. (p.636)

parallel-plate capacitor. Find $\vec{E}$ between the plates



$$\Phi = \Phi_{Top} + \Phi_{bottom} + \Phi_{side}.$$

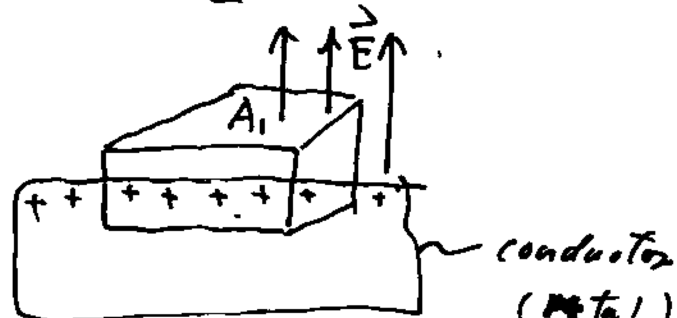
$$= E \cdot A + \underset{E_0}{\underset{\uparrow}{0}} + \underset{\vec{E} \perp \vec{A}}{\underset{\uparrow}{0}}.$$




$$\Phi = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$



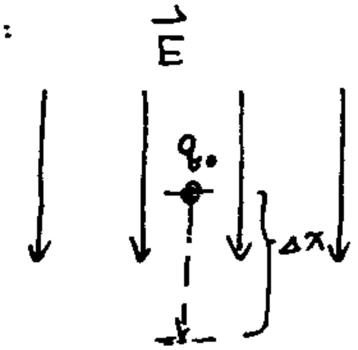
- Electric Potential energy

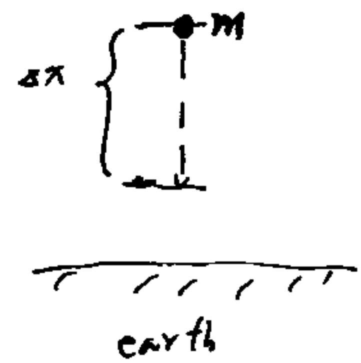
— Electric force is a conservatime force. (Similar to gravitational force)

Electric potential Energy, $U$: $(-\Delta U = W)$

$$\Delta U = -W$$

e.g.:




$$\Delta U = -q_0 E \cdot \Delta x.$$
$$\left(= -q_0 \vec{E} \cdot \vec{\Delta x}\right)$$

$$\Delta U = -mg \cdot \Delta x$$
$$\left(= -m\vec{g} \cdot \vec{\Delta x}\right)$$

- Electric potential $V$

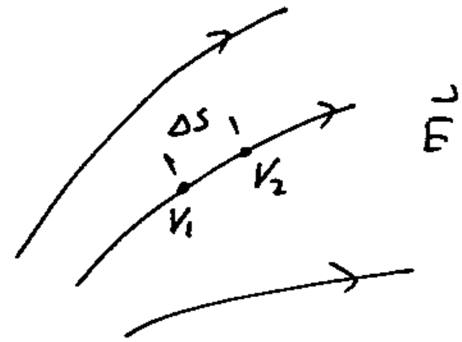
$$\Delta V = \frac{\Delta U}{q_0} = \frac{-W}{q_0}.$$ (independent of $q_0$) (It's a poperty of the field)

$V$ — unit: J/C. (potential energy per charge)

In the e.g.: $\Delta V = \underbrace{-E \cdot \Delta x}_{\text{special. } (\vec{E} /\!/ \vec{\Delta x})}$ $\left(= \underbrace{-\vec{E} \cdot \vec{\Delta x}}_{\text{general.}}\right)$

- Relation between E and V:

along an $\vec{E}$ line




$$\boxed{\begin{aligned} \Delta V &= - E \cdot \Delta S \\ E &= - \frac{\Delta V}{\Delta S} . \end{aligned}}$$

← can be measured.

OR:

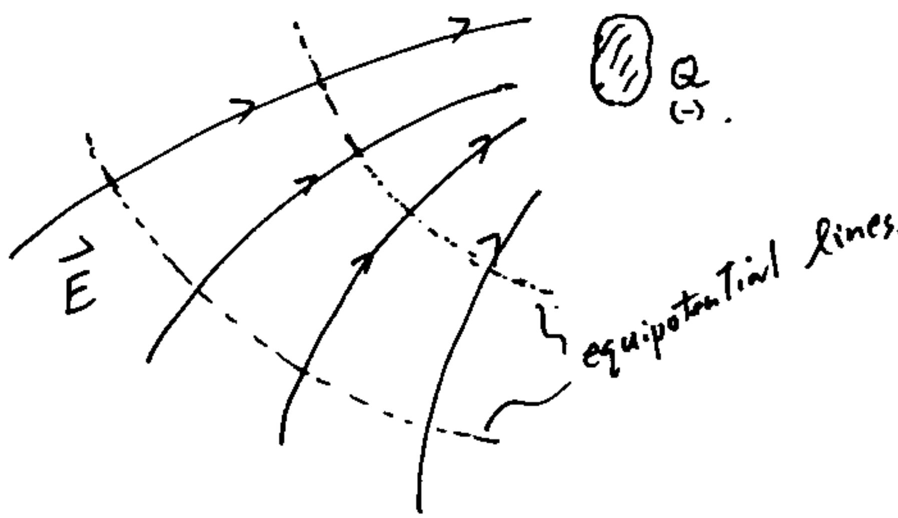
$$E = - \frac{dV}{dS}$$ — can be calculated.

- Equipotential lines (surfaces).

Along an equipotential line, $V$ = same

If a charge is moved along it, No work done by $\vec{E}$-field. ~~i.e., No electric force is acting on it.~~

∴ equipotential line ⊥ $\vec{E}$ field line at any point.
(OR. $\vec{E} = 0$)



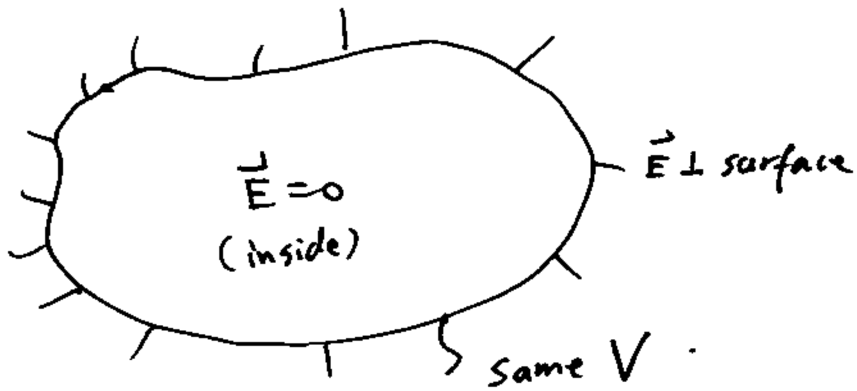
- Ideal conductors are equipotential bodies.

  ↑ charges can move freely.

  (No work done by $\vec{E}$-field).

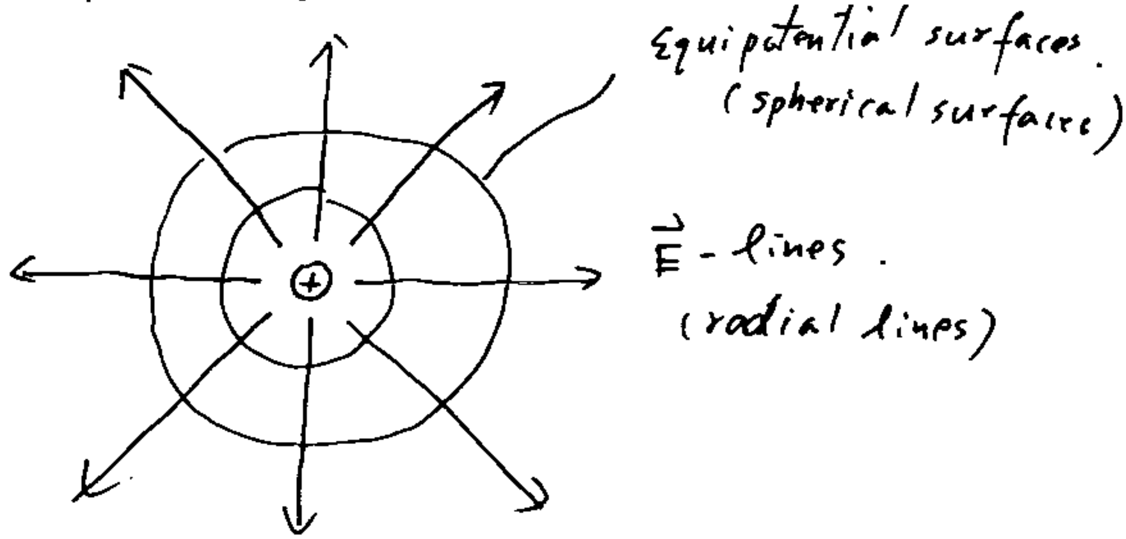
  — inside : $\vec{E} = 0$

  — on surface : $\vec{E} \perp$ surface




- examples of equipotential lines (and field lines) (p.657 -659)

  — A point charge




  — parallel-plate capacitor