Physics 102.

Lecture 12.

- Alternating Currents

$I(t)$, $V(t)$ — time dependent

can be described by sinusoidal functions

e.g.: $$V = A_V \cos(\omega t + \varphi_V)$$

$$I = A_I \cos(\omega t + \varphi_I)$$

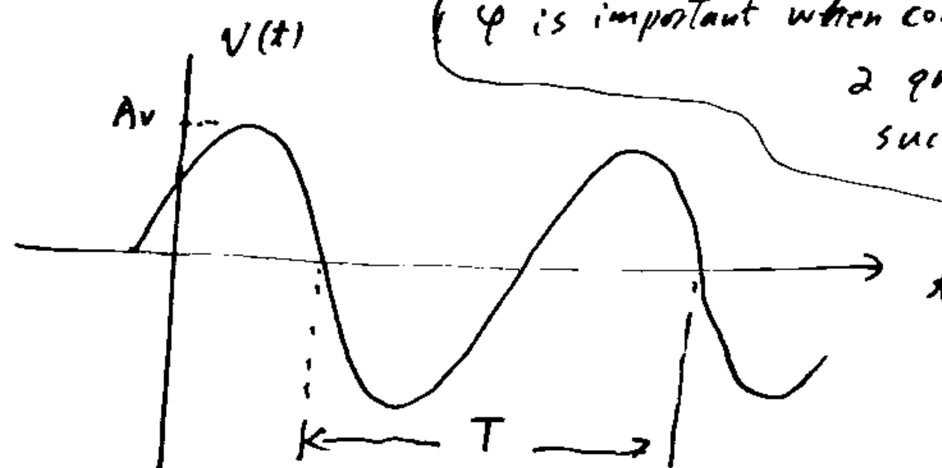
ω — angular frequency ($\omega = 2\pi f$. f - frequency)

A — amplitude

φ — initial phase (phase shift — relative to $\varphi = 0$).

(φ is important when comparing 2 quantities such as $V(t)$, $I(t)$)

e.g.:



$T = \frac{1}{f}$

- e.g.: line voltage in Vancouver:

  $f = 60\ Hz$, $\omega = 2\pi f = 120\pi$ (rad/sec).

  $A = 155$ Volts. (NOT 110 volts).

- what is 110 volts?

  — RMS value (Root Mean square).

$$V_{rms} = \sqrt{(V^2)_{ave}} = \frac{A_v}{\sqrt{2}} \approx 70\%\ A_v.$$

Math:

$$(V^2)_{ave} = \frac{\int_0^T V(t)\,dt}{T}.$$

$$= \frac{\int_0^T A_v^2 \cdot \cos^2(\omega t)\,dt}{T}$$

$$= \frac{A_v^2}{2}.$$

$$\therefore\ V_{rms} = \sqrt{\frac{A_v^2}{2}} = \frac{A_v}{\sqrt{2}}.$$

- Why do we use the RMS value to describe the level of an ac voltage?

— for comparison with DC voltage in terms of power.

$$\text{power} = VI = \frac{V^2}{R} = I^2 R .$$

The average power of ac: $\cancel{P_{ave} \propto (V^2}$

$$P_{ave} = (V^2)_{ave} / R = \frac{V_{rms}^2}{R} = I_{rms}^2 R .$$

DC power: $P = \frac{V^2}{R} = I^2 R .$

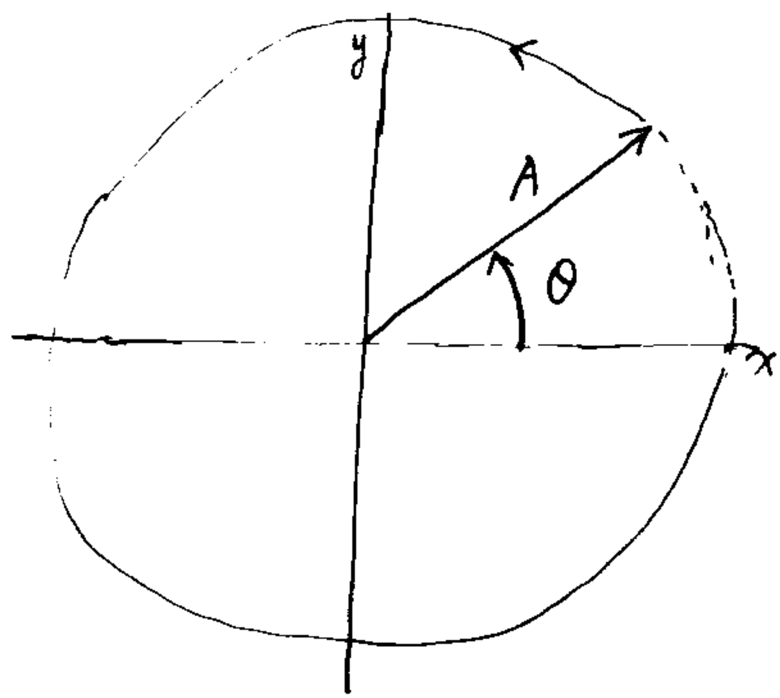
$\therefore$ The line voltage with $V_{rms} = 110$ volts delivers the same power as a DC voltage $V_{DC} = 110$ volts.

- Phasor :

In an ac circuit, ω is the same everywhere. We only need A and φ to describe an AC voltage or current.

Idea: $x = A\cos(\omega t + \varphi)$

can be graphically described by a rotating Vector $\vec{A}$:



$\theta = \omega t + \varphi$

The x-component:

$x = A \cdot \cos\theta$.

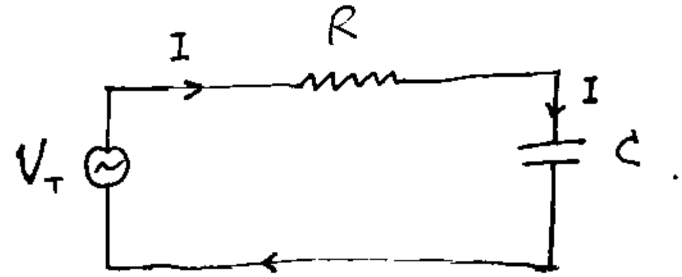
when $\vec{A}$ is rotating at a constant angular velocity ω and with an initial angle φ:

$\theta = \omega t + \varphi$

Then: $x = A \cdot \cos(\omega t + \varphi)$

(x can be $V(t)$ or $I(t)$)

e.g.: an RC circuit:



— I is the same everywhere (because of the series connection).

$\therefore$ choose the phase of I to be 0. ($\varphi_I = 0$)

$I = A_I \cdot \cos(\omega t)$ (use it as the phase reference).

— The voltage across R:

$$V_R = I R = R A_I \cos(\omega t)$$

$\therefore$ V_R and I_R are in phase.

— The voltage across C: (NOT simple multiplication !)

$$I = \frac{dQ}{dt} = C \cdot \frac{dV_c}{dt} \qquad (Q = C V_c).$$

Then: $\dfrac{dV_c}{dt} = \dfrac{1}{C} \cdot A_I \cdot \cos(\omega t)$.

$$V_c = \frac{1}{\omega C} A_I \sin(\omega t) = \frac{1}{\omega C} A_I \cdot \cos\left(\omega t - \frac{\pi}{2}\right)$$
$$= A_{V_c} \cdot \cos(\omega t + \varphi_{V_c})$$

$\therefore$ $\varphi_{V_c} = -\frac{\pi}{2}$. $\qquad A_{V_c} = \frac{1}{\omega C} A_I$.

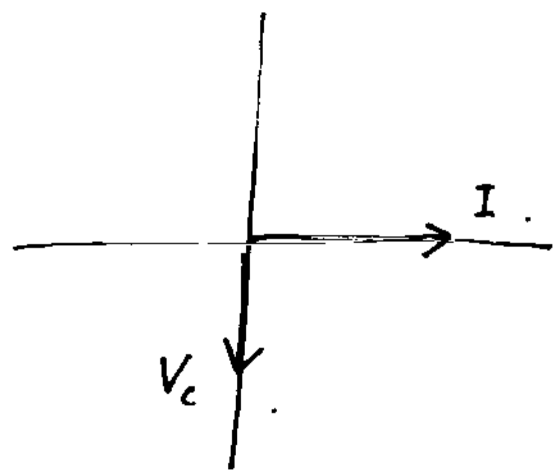
Meaning of $\varphi_{V_c} = -\frac{\pi}{2}$:

There is a phase shift between the current and voltage of the capacitor !

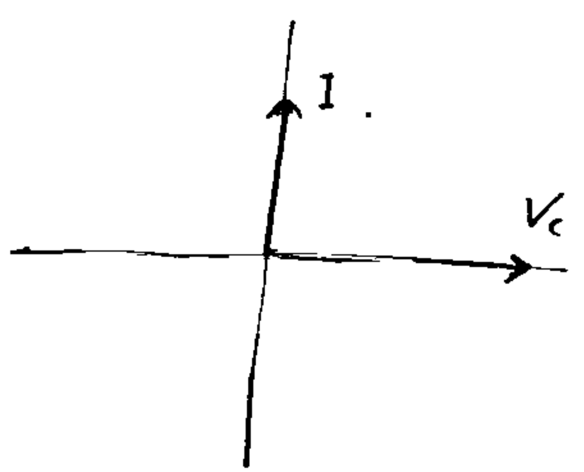
i.e. the voltage lags by $\frac{\pi}{2}$.

OR the current leads by $\frac{\pi}{2}$.

In a phasor representation :



OR .



(" ICE ")

for a capacitor .

I goes first, V is behind.
(E)

— define $X_c = \frac{1}{\omega C}$

as the reactance of the capacitor.

(just like the resistance of a resistor).

$V_{max} = X_C \cdot I_{max}$. (~~factor~~ for capacitors)

— The voltages V_R, V_c and V_T.

In the phasor diagram, use I as the reference.

$V_T = V_R + V_c$.

Vector sum!