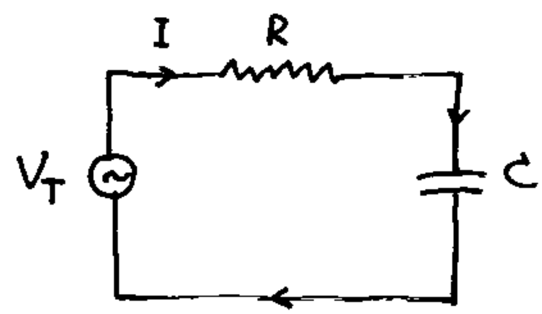
Physics 102

Lecture 13

- RC circuit as a low pass filter



We are interested in the relationship among $I(t)$, $V_R(t)$, $V_C(t)$, $V_T(t)$.

I — common, choose $\varphi_I = 0$ as phase reference.

$$I = A_I \cos(\omega t)$$

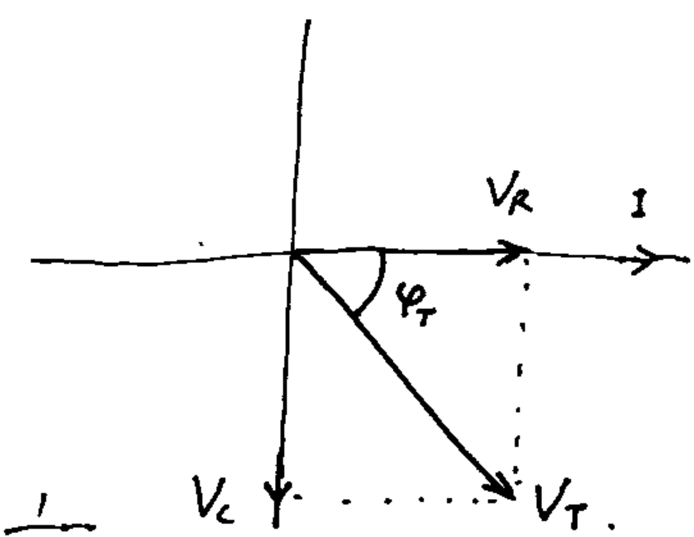
$$V_R = A_{VR} \cdot \cos(\omega t) \qquad A_{VR} = R \cdot A_I$$

$$V_C = A_{VC} \cdot \cos(\omega t - \tfrac{\pi}{2}), \qquad A_{VC} = \frac{1}{\omega C} \cdot A_I = X_C \cdot A_I$$

$$V_T = A_{VT} \cdot \cos(\omega t + \varphi_T)$$

Since $V_T = V_R + V_C$

In the phasor diagram, this is a vector sum.



$$\therefore \quad A_T = \sqrt{A_{VR}^2 + A_{VC}^2}$$

$$\varphi_T = -\tan^{-1}\frac{A_{VC}}{A_{VR}} = -\tan^{-1}\frac{1}{\omega RC}.$$

- low- and high- frequency behaviours.

(Due to $X_C = \frac{1}{\omega C}$)

capacitor has a small reactance at high frequencies.

e.g. when $\omega = 0$, it's DC.

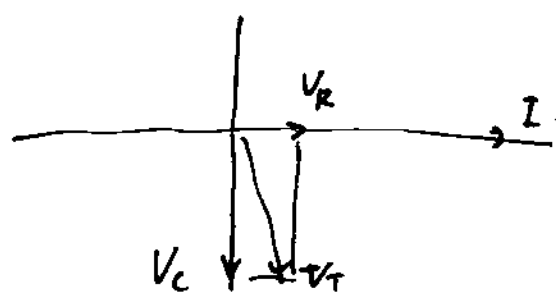
$X_C \to \infty$. the current cannot get through continuously. because it will stop when the capacitor is fully charged.

In the RC circuit,

low-f: when $\frac{1}{\omega C} >> R$, i.e, $\omega << \frac{1}{RC}$.

amplitudes: $V_C >> V_R$.

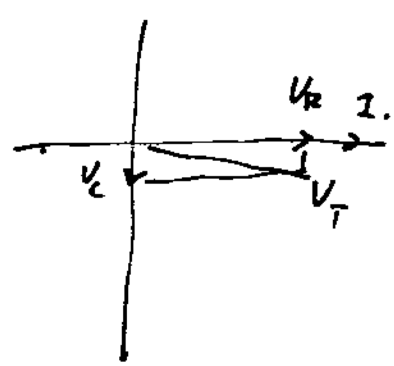
$I \cdot \frac{1}{\omega C}$ IR.



i.e. $V_C \approx V_T$. $\because V_T = V_C + V_R$.

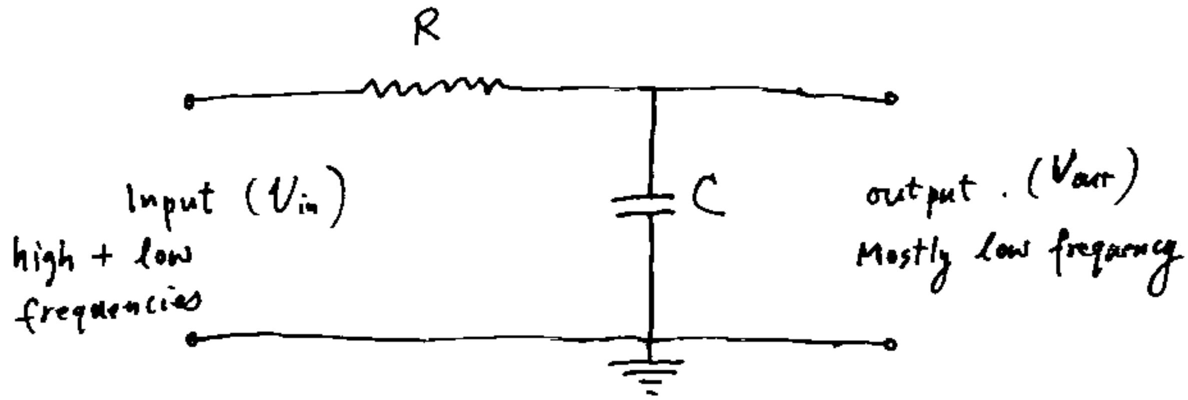
high-f: when $\frac{1}{\omega C} << R$. i.e. $\omega >> \frac{1}{RC}$.

$V_R >> V_C$.



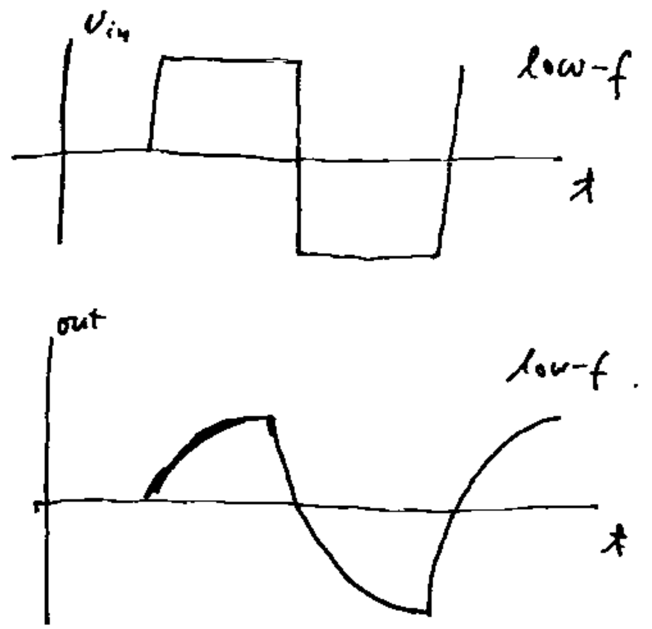
$V_T \approx V_R$, $V_C << V_T$.

• So, we can use the RC circuit as a low pass filter.



why? (physically).

Explanation: It takes time to charge/discharge.



★ at low-freq:

We have long enough time to charge up the capacitor.

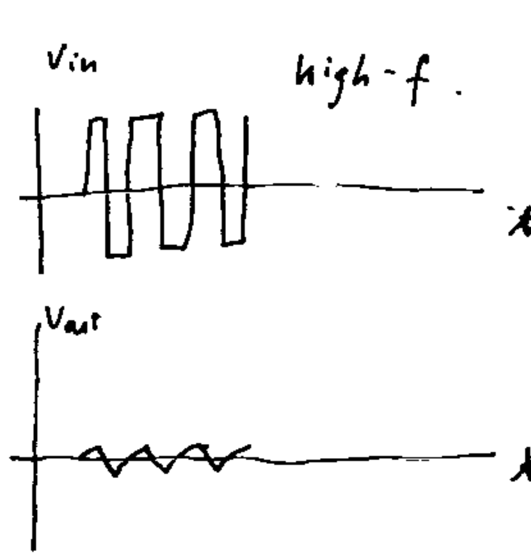
~~before it~~

★ at high-freq.

The driving signal V_{in} change direction quickly.

the capacitor doesn't get enough time to charge up.

i.e, the voltage across C can only go up a little, then the driving voltage starts to force it to discharge.



∴ $|V_{out}| \ll |V_{in}|$

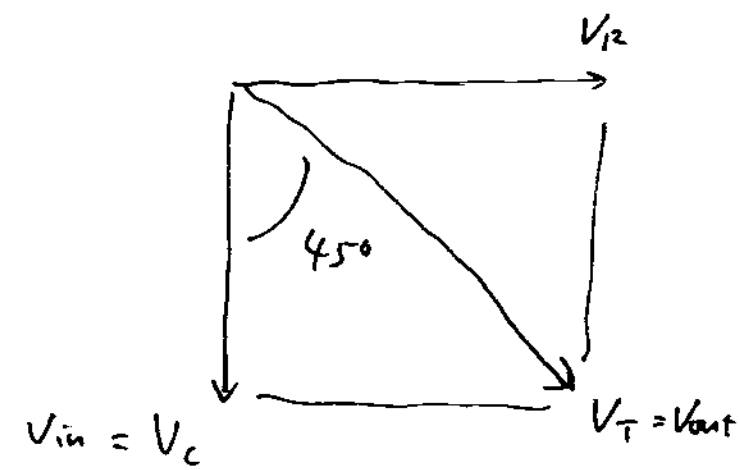
- Cut off freq. $\omega_c = \frac{1}{RC}$. (rad/sec).

  $f_c = \frac{1}{2\pi RC}$. (Hz).

at cut off frequency. $\frac{1}{\omega C} = R$.

amplitude: $V_C = V_R$

$|V_{out}| = \frac{|V_{in}|}{\sqrt{2}}$.

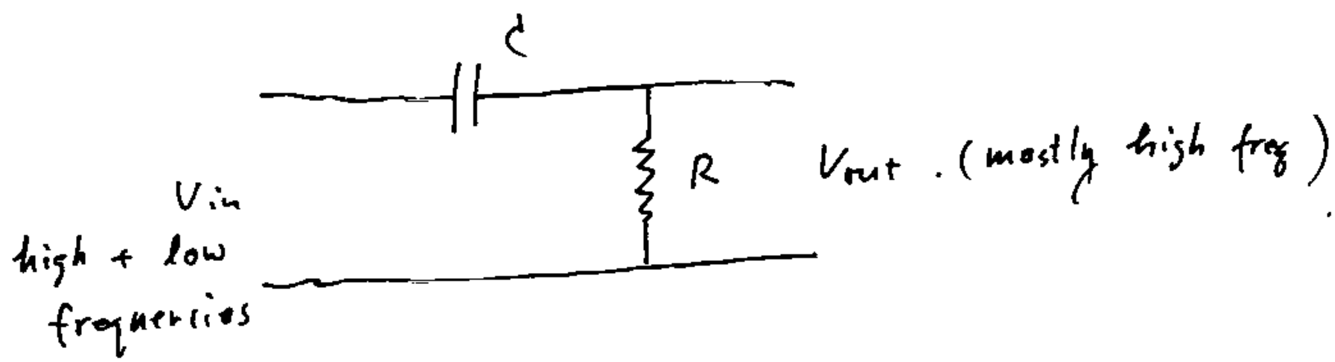


phase:

45° phase shift

between V_{in} and V_{out}.

- An RC circuit can be used as a high-pass filter too.



Same analysis as the low-pass filter.

Next: Magnetism. ch.22.