

Lecture 20 .

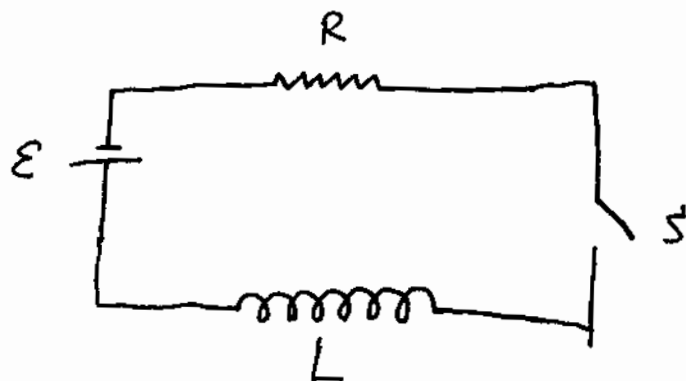
Wed. Oct. 27, 2004

• RL Circuits .

Induced emf:

$$\mathcal{E}_L = -N \frac{d\Phi}{dt}$$

$$|\mathcal{E}_L| = L \cdot \left| \frac{dI}{dt} \right|$$



Lenz's law . induced current will oppose the change .

$\therefore$  No abrupt change in  $I$  of  $L$

e.g. when you switch on the circuit at time  $t=0$  .

Then, the current will go up gradually .

After a long time ( $t \rightarrow \infty$ ).

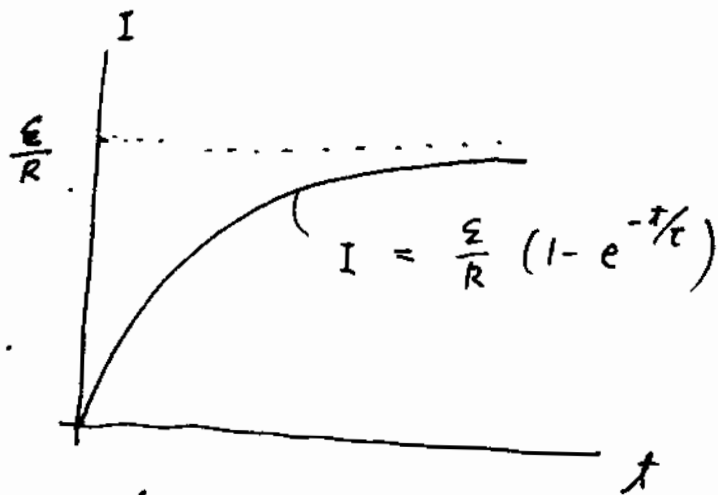
the system is stable .

i.e.  $\frac{dI}{dt} = 0$  .

No more induced current .

$\therefore$   $L$  has no effect .

just behave like a wire . (ideally  $R_L = 0$ ) .



Then .  $I = \frac{\mathcal{E}}{R}$  . ( $t \rightarrow \infty$ ) .

• time dependence of the current in  $L$ :

$$I = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau})$$

where time constant  $\tau$  is:

$$\tau = \frac{L}{R}.$$

The larger the  $L$ , the stronger the inductance effect.

The longer it takes to reach a stable state.

- Energy stored in an inductor

$$U = \frac{1}{2} L I^2 .$$

In fact, the energy is stored ~~in~~ <sup>in</sup> the  $\vec{B}$ -field inside the inductor.

magnetic energy density:

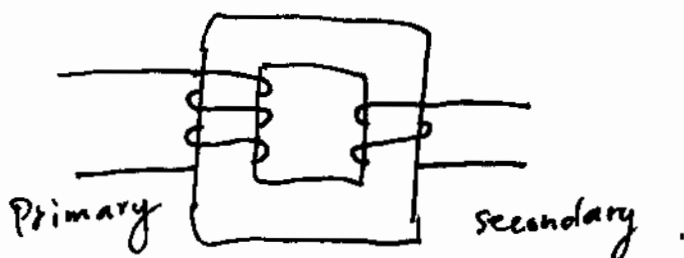
$$u_B = \frac{B^2}{2\mu_0} .$$

Note: electric energy density: (e.g. inside a capacitor)

$$u_E = \epsilon_0 E^2 / 2 .$$

- Transformer:

$$\frac{\mathcal{E}_p}{\mathcal{E}_s} = \frac{N_p}{N_s} .$$



OR:

$$\boxed{\frac{V_p}{V_s} = \frac{N_p}{N_s} .}$$

Why?

~~you work it out!~~  
you work it out!

Problem. \*44. of ch.23.

A solenoid.  $N, A, \ell$ .

when  $\frac{dI}{dt} = 2.0 \text{ A/s}$ ,  $\mathcal{E} = 75 \text{ mV}$

(a).  $L = ?$

$$|\mathcal{E}| = L \left| \frac{dI}{dt} \right|$$

$$L = \frac{|\mathcal{E}|}{\frac{dI}{dt}} = \frac{75 \text{ mV}}{2 \text{ A/s}} = 3.75 \times 10^{-2} \text{ H.}$$

$$\approx 38 \text{ mH.}$$

(b).  $N, A$  } unchanged, but  $\ell$  is doubled.  
 $\frac{dI}{dt}$  } ( $\ell' = 2\ell$ ).

What would happen to  $|\mathcal{E}|$ ?

For a solenoid,  $L = \mu_0 n^2 A \ell = \frac{\mu_0 N^2 A}{\ell}$ .

When  $\ell$  is doubled.

$L$  will be ~~double~~ halved.

$\therefore |\mathcal{E}| = L \left| \frac{dI}{dt} \right|$  will be halved.

$$\therefore |\mathcal{E}| < 75 \text{ mV.}$$

(c).  $|\mathcal{E}| = \frac{1}{2} \cdot 75 \text{ mV} \approx 38 \text{ mV.}$

Problem #77 of Ch. 23.

① Motion of rod.

- (a) just released,  $v=0$ .  
 No magnetic force.  
 $a=g$ .

Then,  $v \neq 0$ .

$$I \neq 0, \quad I = BLv/R$$

$$\text{Magnetic force: } F_B = BIL \\ = \frac{B^2 L^2 v}{R}$$

against the motion.

$$F_B = -\frac{B^2 L^2 v}{R}$$

just like drag force on  
 an falling object in air.

The speed of falling will  
 keep increasing until.

$$\frac{B^2 L^2 v}{R} = mg$$

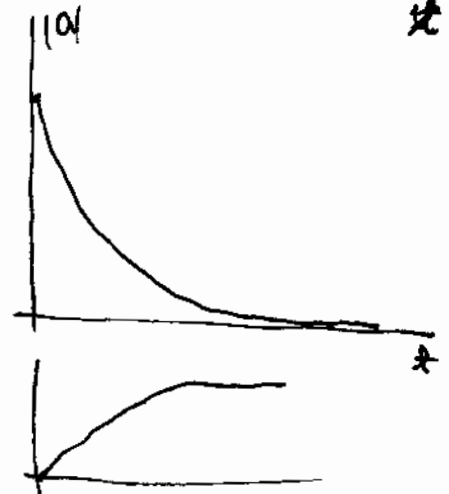
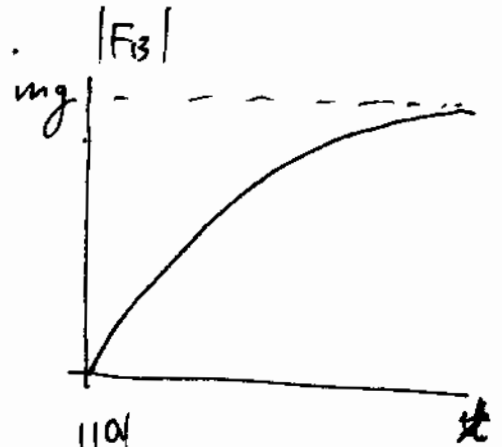
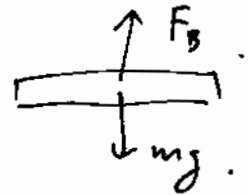
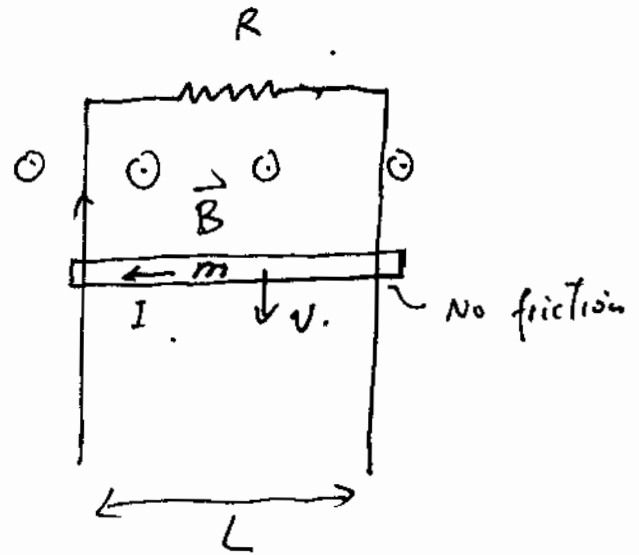
Then,  $F_{\text{net}} = 0$ .

$$a = 0$$

$$v = v_{\text{Terminal}}$$

The rod will eventually ~~fit~~ reach a  
 constant speed of falling.

↪ terminal velocity.

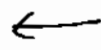


(b). Direction of current.

RHR.



OR Lenz's law.



(c). at terminal speed:

$$F_{net} = 0.$$

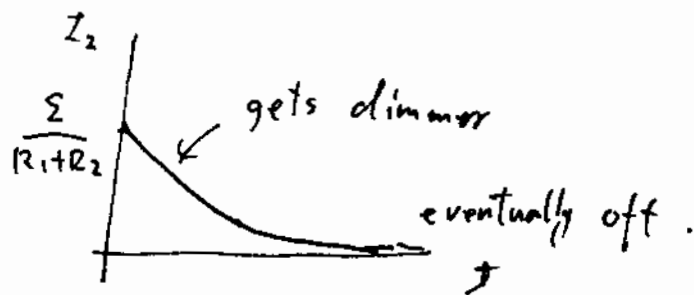
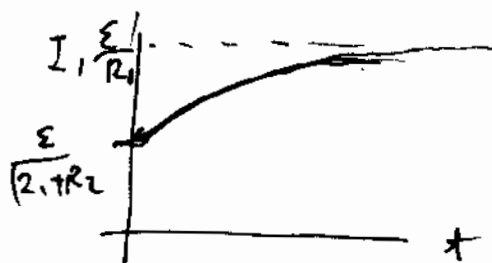
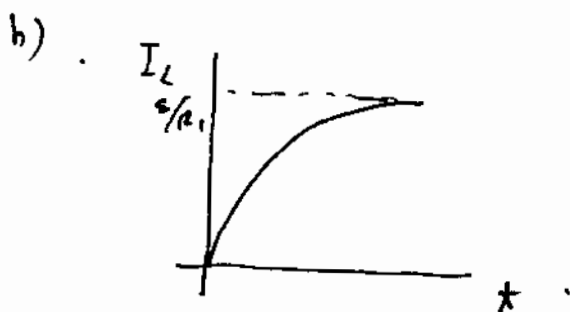
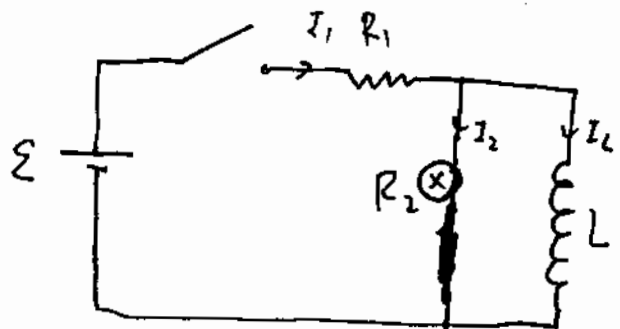
$$F_B - mg = 0.$$

$$\frac{B^2 L^2 v}{R} = mg.$$

$$v = \frac{mgR}{B^2 L^2}.$$

79. a).  $x \rightarrow \infty$   $L$  is like a resistor  $R_L = 0$ .

$$I = \left( \frac{\mathcal{E}}{R_1 + R_2} \right) \quad I = \frac{\mathcal{E}}{R_1}$$



c). induct' e.m.f.

d).  $V_{before} = 0$

$$V_{after} = IR_2 = \left( \frac{\mathcal{E}}{R_1} \right) R_2$$