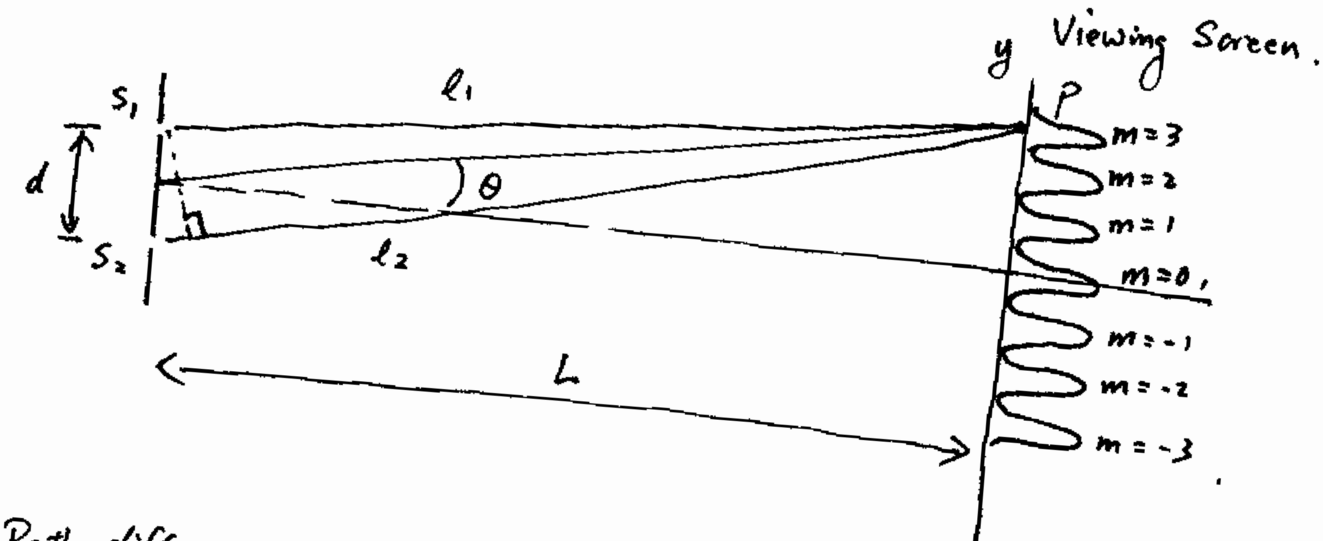
Physics 102

Lecture 31.

Wed. Nov. 24, 2004

- Two-slit Interference



- Path difference: $\Delta l = l_2 - l_1 = d \cdot \sin\theta$

- Constructive interference: (Bright fringes.)

$$d \cdot \sin\theta = 0, \pm\lambda, \pm 2\lambda, \ldots$$

$$d \cdot \sin\theta = m\lambda \quad (m = 0, \pm 1, \pm 2, \ldots)$$

- Destructive interference: (Dark fringes)

$$d \cdot \sin\theta = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}, \pm\frac{5\lambda}{2}, \ldots$$

$$d \cdot \sin\theta = (m - \tfrac{1}{2})\lambda \quad (m = 0, \pm 1, \pm 2, \ldots)$$

- Location of mth bright fringe on the viewing screen:

$$y_m = L \cdot \tan\theta_m \quad \text{where} \quad \tan\theta_m \approx \sin\theta_m \; (\text{small } \theta)$$

$$\therefore \quad y_m \approx L \cdot \frac{m\lambda}{d}. \qquad \approx \frac{m\lambda}{d}.$$

- fringe spacing.

$$\frac{y_m}{m} \approx L \cdot \frac{\lambda}{d} \qquad \propto \frac{1}{d}.$$

The smaller the d, the larger the $\frac{y_m}{m}$. (fringe spacing).

When d is increased, fringe spacing will be decreased.

Demo (movie): (water wave interference)

When S_1 and S_2 are separated by a larger distance we get a finer interference pattern.

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e.g: Two narrow slits separated by 1.5 mm are illuminated by sodium light of wave length 589 nm. Interference fringes are observed on a screen 3 m away. What is the fringe spacing on the screen?

[Solution]: mth bright fringe: $d \cdot \sin\theta_m = m\lambda$.

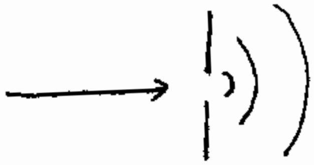
$$y_m = L \cdot \tan\theta_m.$$

for small θ, $\sin\theta \approx \tan\theta$.

$$\therefore \; y_m \approx L \cdot \sin\theta_m = L \cdot \frac{m\lambda}{d}.$$

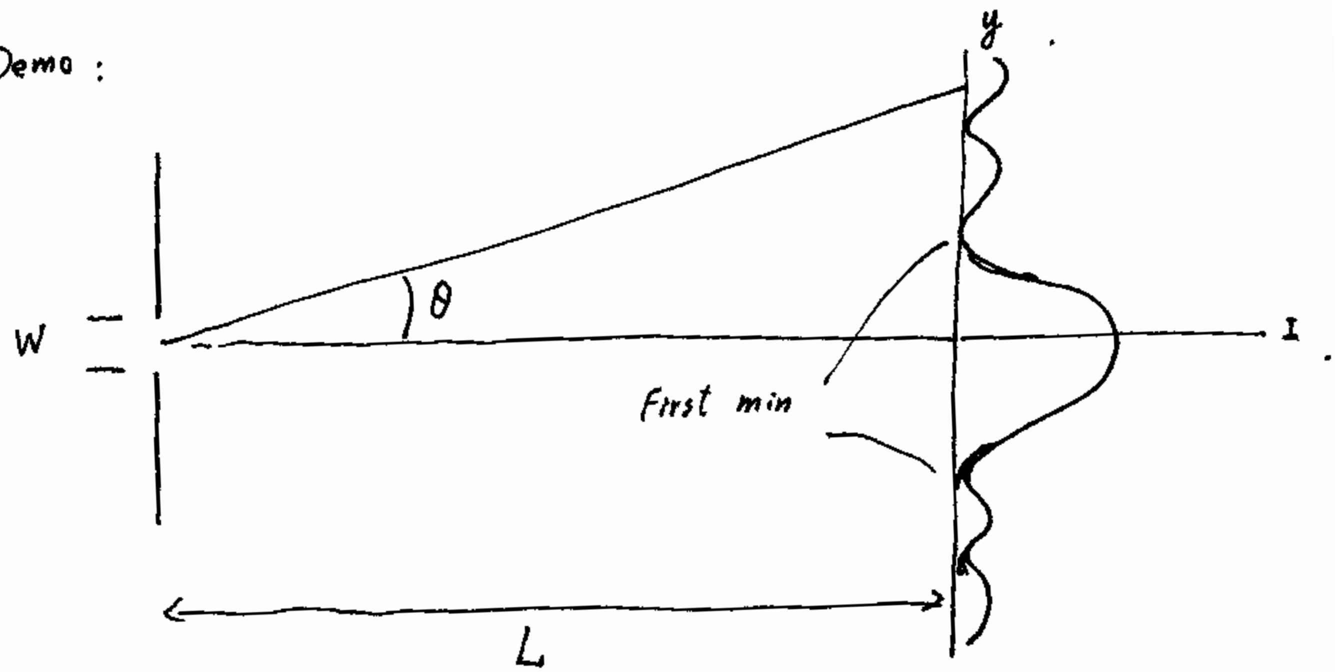
Fringe spacing: $\frac{y_m}{m} = L \cdot \frac{\lambda}{d} = \frac{589\times10^{-9}\times 3}{0.0015} = 1.18\text{ mm}.$

- Single-Slit Diffraction



wave — (Not a straight line like a ray of particles)

Demo :



- Central Maximum : $\theta = 0$.

- Dark fringes : $W \cdot \sin\theta = m\lambda$, $m = \pm 1, \pm 2, \pm 3, \cdots$

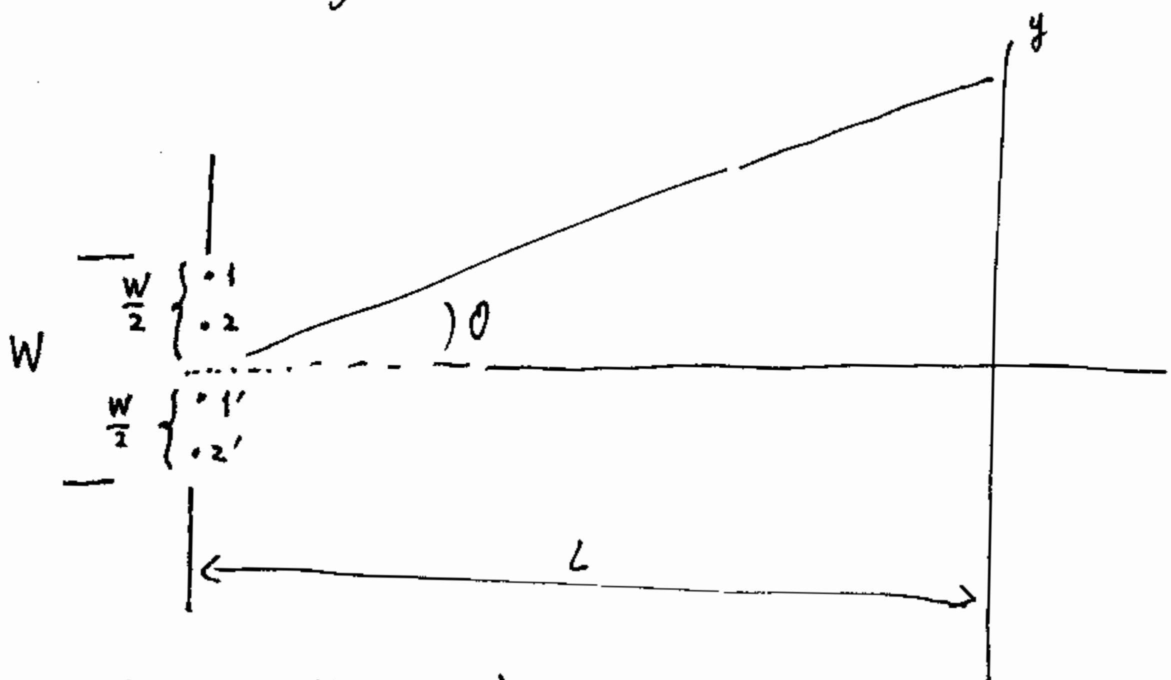
  First minimum : $W \cdot \sin\theta = \lambda$.

  $$y = L \cdot \tan\theta \approx L \cdot \frac{\lambda}{W} \quad (\theta \text{ — small}).$$

- Explanation :

  The slit is a distribution of point sources. The light from different points at the slit will interfere with each other on the screen.

- First minimum of a single-slit diffraction pattern.



pairs : (1, 1') (2, 2') .....

1 and 1' interfere just like S and S' in a 2-slit setting.

The distance between 1 and 1' is $\frac{W}{2}$.

∴ ~~Dark~~ First Dark fringe : $\frac{W}{2} \sin\theta = \frac{\lambda}{2}$.

$$\therefore \quad W \cdot \sin\theta = \lambda .$$

- For second and higher fringes, we would need to analyze the contributions from all the sources in a fairly tedious way.

m th dark fringe : $W \cdot \sin\theta = m\lambda \quad (m = \pm 1, \pm 2, \dots)$

- Position of first minimum on the viewing screen.

$$y = L \cdot \tan\theta \approx L \cdot \sin\theta \qquad (\theta - \text{small})$$

$$= \frac{L \cdot \lambda}{W}. \quad \propto \frac{1}{W}.$$

Demo: The narrower the slit width

The wider the central bright area.

- e.g.: $\lambda = 700\ nm. = 7 \times 10^{-7}\ m$

$W = 0.2\ mm = 2 \times 10^{-4}\ m.$

$L = 6\ m.$

Q: What is the distance between the first minimum on the left and the first minimum on the right of the central maximum?

$$y = \frac{L\lambda}{W} = \frac{6 \times 7 \times 10^{-7}}{2 \times 10^{-4}} = 2.1 \times 10^{-2}\ m = 2.1\ cm.$$

The width of the central maximum:

$$2y = 4.2\ cm.$$