PHYS 100 Midterm examination #1 (vers. 1A) Key

Feb. 9, 2007 Name_____

Time: 50 minutes Student No. Blue

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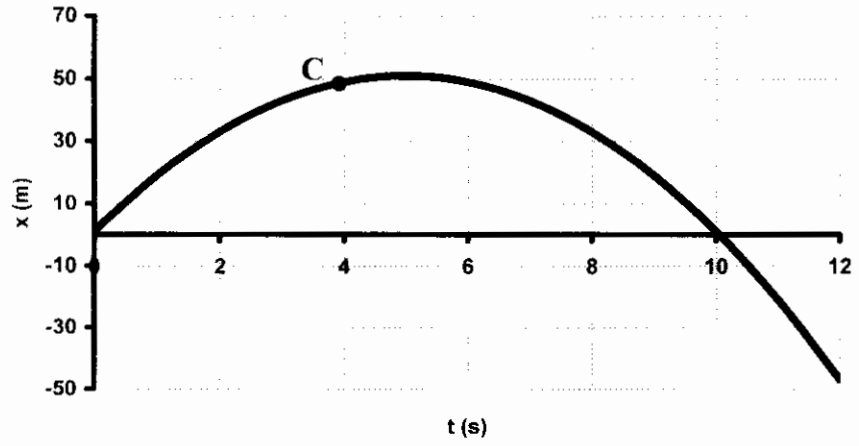
Part I (Multiple choice questions). For each of the following five questions, please circle one answer only.

CDCEAB .

Please refer to the following position-versus-time graph when answering questions 1-3:



1$_{(1/20)}$. The average velocity over the interval between t=0 and t=5 s is

A) –4 m/s
B) 0
(C) 10 m/s
D) 0.1 m/s
E) –10 m/s

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{50 - 0\ (m)}{5 - 0\ (s)} = 10\ m/s$$

2$_{(1/20)}$. The instantaneous velocity at point C is

A) Greater than the average velocity over the interval t=0 to t=5 s
B) Negative
C) Equal to the average velocity over the interval t=0 to t=5 s
(D) Smaller than the average velocity over the interval t=0 to t=5 s
E) Increasing

3$_{(1/20)}$. The displacement during the time interval between t=5 and t=10 s is

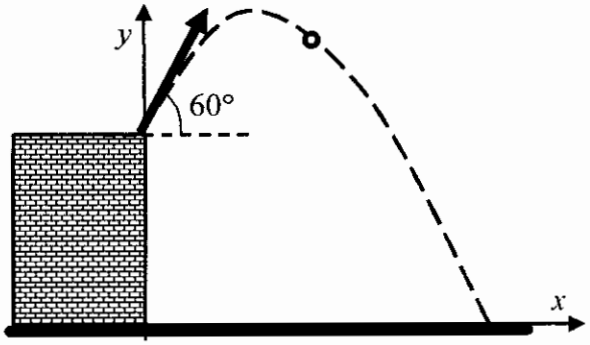
A) Increasing
B) 50 m
(C) –50 m
D) 50 m/s
E) 5 m

$$\Delta x = x_2 - x_1 = 0 - 50\ (m) = -50\ m$$

4$_{(1/20)}$. A satellite moves around the earth in a circular orbit with a constant speed.

A) The velocity of the satellite increases.
B) The velocity of the satellite decreases.
C) The velocity of the satellite does not change.
D) The acceleration of the satellite is constant.
(E) The acceleration of the satellite is not constant.

5$_{(1/20)}$. A ball is thrown from a roof at a 60° angle above the horizontal with a speed of 20 m/s. 6.0 seconds later, the ball hits the ground. Ignore the air resistance.



When the ball hits the ground, the x-component of the velocity is

(A) 10m/s.
B) 41m/s.
C) 76m/s.
D) 17m/s.
E) 39m/s.

$a_x = 0$.

$v_x = v_{x0} = v_0 \cos\theta$

$= 20 \cdot \cos 60°$

$= 10$ m/s

6$_{(1/20)}$. (refer to the previous question) When the ball hits the ground, the y-component of the velocity is

A) -10m/s.
(B) -41m/s.
C) -76m/s.
D) -17m/s.
E) -39m/s.

$a_y = -g$. $v_y = v_{y0} + a_y t$

$= v_0 \sin\theta - gt$

$= 20 \cdot \sin 60° - (9.8)(6) = -41$ m/s

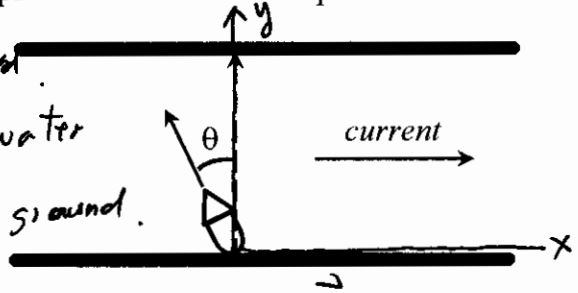
Part II (Full solution questions).

7$_{(4/20)}$. The current of a river is flowing at 4 km/h. A man wants to drive a boat across the river to reach a point on the other bank directly opposite to his starting point. His boat is capable of travelling 5 km/h in still water. At what angle upstream should the man point his boat?

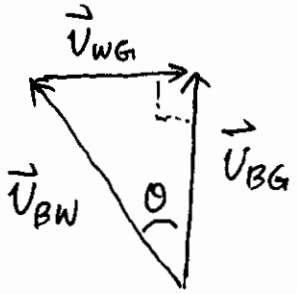
$\vec{v}_{BG}$ — velocity of Boat relative to ground.

$\vec{v}_{BW}$ — velocity of Boat relative to water

$\vec{v}_{WG}$ — velocity of water relative to ground.



$\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$



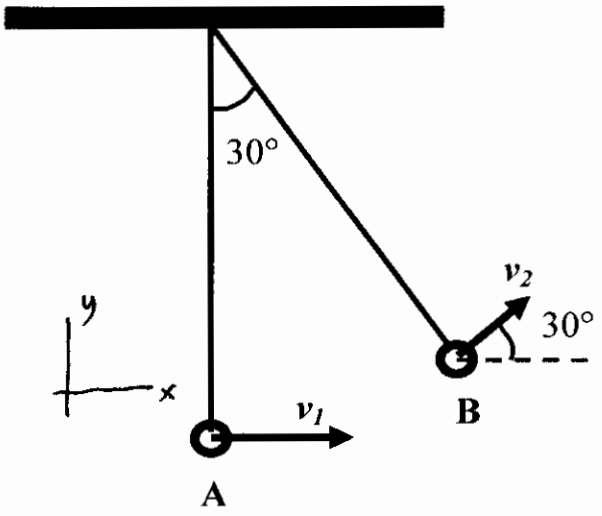
$\theta = \sin^{-1} \frac{v_{WG}}{v_{BW}} = \sin^{-1} \frac{4 \text{ km/h}}{5 \text{ km/h}}$

$= \sin^{-1} 0.8$

$= 53°$.

$8_{(5/20)}$. A pendulum is a small bob swinging to and fro on a string, as shown in the figure below. The bob reaches the bottom A with a horizontal velocity of 2.4m/s to the right. 0.50s later, it reaches B with a velocity 0.80m/s at an angle of 30°. Find the average acceleration of the bob over the time interval between A and B.

Set up coordinate system: y ↑, x →



Given: when $t_i = 0$

$v_{Ax} = 2.4$ m/s

$v_{Ay} = 0$

when $t_f = 0.50$ s.

$v_{Bx} = v_2 \cos 30°$

$= 0.69$ m/s

$v_{By} = v_2 \sin 30°$

$= 0.4$ m/s.

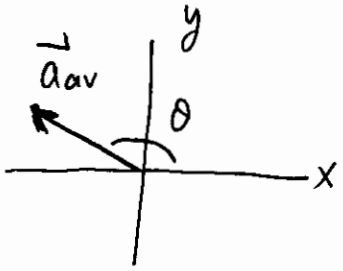
Average acceleration: $\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_B - \vec{v}_A}{\Delta t}$

x-component: $a_{avx} = \frac{v_{Bx} - v_{Ax}}{\Delta t} = \frac{0.69 - 2.4}{0.5} = -3.42$ m/s²

y-component: $a_{avy} = \frac{v_{By} - v_{Ay}}{\Delta t} = \frac{0.4 - 0}{0.5} = 0.8$ m/s²

Magnitude: $a_{av} = \sqrt{(a_{avx})^2 + (a_{avy})^2} = \sqrt{(-3.42)^2 + (0.8)^2} = 3.51$ m/s²

Direction: ($a_{avx} < 0$, $a_{avy} > 0$



$\theta = \tan^{-1} \frac{a_{avy}}{a_{avx}} = 167°$

$9_{(5/20)}$. John is driving a van on a single-lane section of the Sea-to-Sky highway. In order to pass a bus he crosses the yellow line and speeds up to 120 km/h. At the same time, a truck is coming in the opposite direction with a speed of 90 km/h. When the two vehicles are 100 m apart, both drivers hit the brakes, which results in a constant deceleration of 6.0 m/s^2 for both vehicles.

A) Draw the v-t and x-t graphs to determine whether or not the two vehicles will collide.

B) If the two vehicles collide, find their relative velocity at the instant of collision; if they don't collide, find the distance between the two vehicles after they have both come to a complete stop.

[Solution]: set up coordinate system.

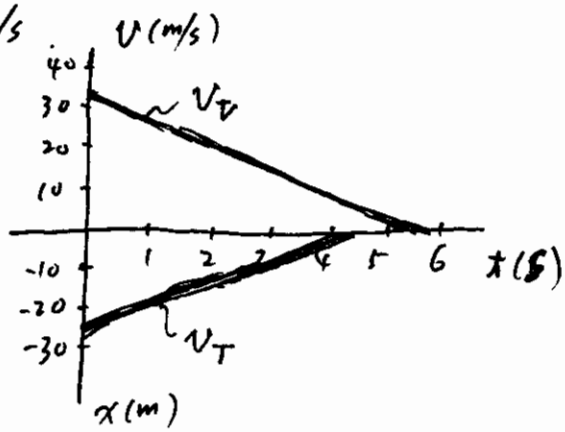
Given: when $t=0$, $v_{V0} = 120\,km/h = 33.3\ m/s$.

$v_{T0} = -90\,km/h = -25\ m/s$

$x_{V0} = 0$

$x_{T0} = 100\ m$.

at t: $a_V = -6\ m/s^2$, $a_T = 6\ m/s^2$.

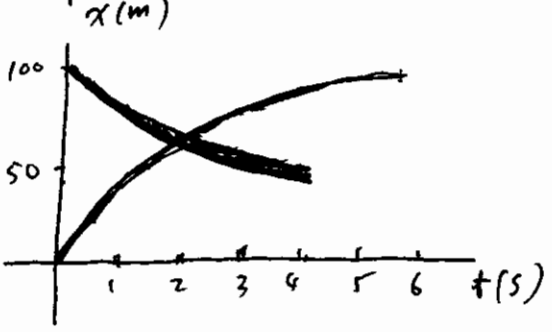


Van: $v_V = v_{V0} + a_V t = 33.3 - 6t$

Truck: $v_T = v_{T0} + a_T t = -25 + 6t$

position: $x_V = 33.3t - \frac{1}{2}\cdot 6\cdot t^2 = 33.3t - 3t^2$

$x_T = 100 - 25t + 3t^2$



A). From the x-t graph, the two vehicles will collide at about $t = 2\,s$.

Solve for t: $x_V = x_T$: $100 - 25t + 3t^2 = 33.3t - 3t^2$

$6t^2 - 58.3t + 100 = 0$

$t = 2.2\ s$. ($t = 7.5\ s$ rejected. $> 5.55\ s$)

at $t = 2.2\ s$:

$v_V = 33.3 - 6\cdot(2.2) = 20.1\ m/s$ $\quad v_T = -25 + 6(2.2) = -11.8\ m/s$

relative velocity: $v_r = v_V - v_T = 20.1 + 11.8 = 31.9\ m/s = 115\ km/h$.

PHYS 100 Midterm examination #1 (vers. 1B)

Feb. 9, 2007 Name Key

Time: 50 minutes Student No. Green

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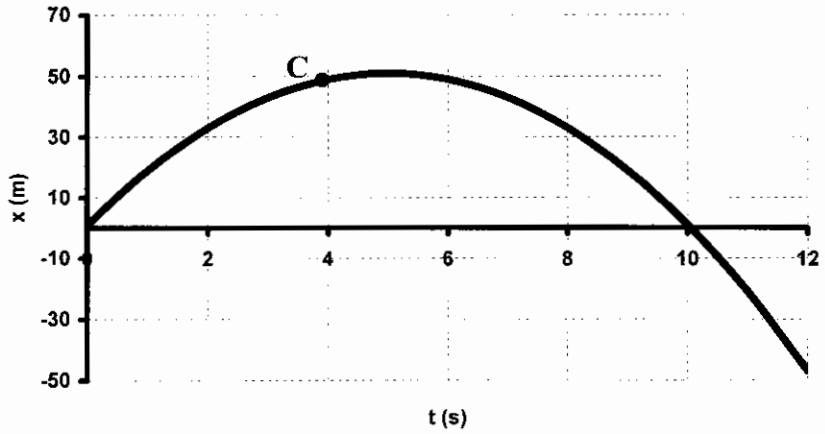
Part I (Multiple choice questions). For each of the following five questions, please circle one answer only.

DCABBE

Please refer to the following position-versus-time graph when answering questions 1-3:



1(1/20). The instantaneous velocity at point C is
- A) Greater than the average velocity over the interval t=0 to t=5 s
- B) Negative
- C) Equal to the average velocity over the interval t=0 to t=5 s
- (D) Smaller than the average velocity over the interval t=0 to t=5 s
- E) Increasing

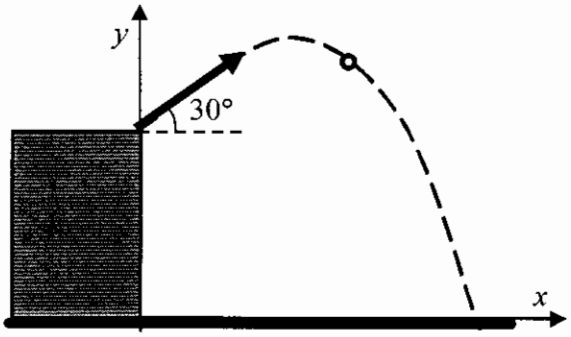
2(1/20). The average velocity over the interval between t=0 and t=5 s is
- A) –4 m/s
- B) 0
- (C) 10 m/s
- D) 0.1 m/s
- E) –10 m/s

3(1/20). The displacement during the time interval between t=5 and t=10 s is
- (A) -50 m
- B) -50 m/s
- C) –5 m
- D) 50 m/s
- E) Increasing

$4_{(1/20)}$. A satellite moves around the earth in a circular orbit with a constant speed.

A) The acceleration of the satellite is constant.
B) The acceleration of the satellite is not constant.
C) The velocity of the satellite does not change.
D) The velocity of the satellite increases.
E) The velocity of the satellite decreases.

$5_{(1/20)}$. A ball is thrown from a roof at a 30° angle above the horizontal with a speed of 20 m/s. 6.0 seconds later, the ball hits the ground. Ignore the air resistance.



When the ball hits the ground, the x-component of the velocity is

A) 10m/s.
B) 17m/s.
C) 76m/s.
D) 41m/s.
E) 49m/s.

$6_{(1/20)}$. (refer to the previous question) When the ball hits the ground, the y-component of the velocity is

A) -10m/s.
B) -17m/s.
C) -76m/s.
D) -41m/s.
E) -49m/s.

Part II (Full solution questions).

$7_{(4/20)}$. The current of a river is flowing at 3 km/h. A man wants to drive a boat across the river to reach a point on the other bank directly opposite to his starting point. His boat is capable of travelling 4 km/h in still water. At what angle upstream should the man point his boat?

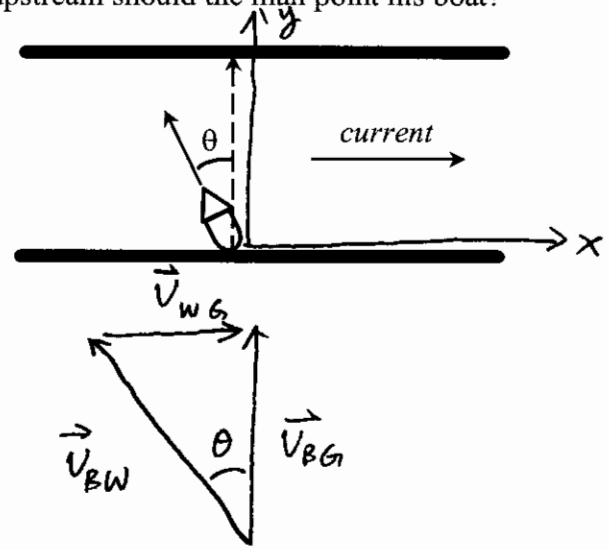
$$\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$$

$$\theta = \sin^{-1}\frac{v_{WG}}{v_{BW}}$$

$$= \sin^{-1}\frac{3}{4}$$

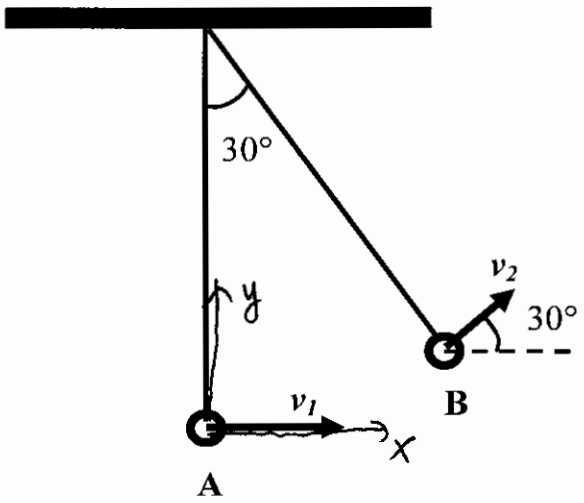
$$=$$

$$\approx 49^\circ$$



$8_{(5/20)}$. A pendulum is a small bob swinging to and fro on a string, as shown in the figure below. The bob reaches the bottom A with a horizontal velocity of 2.5m/s to the right. 0.40s later, it reaches B with a velocity 1.0m/s at an angle of 30°. Find the average acceleration of the bob over the time interval between A and B.

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_B - \vec{v}_A}{0.40\,(s)}$$



$$a_{avx} = \frac{v_{Bx} - v_{Ax}}{\Delta t}$$

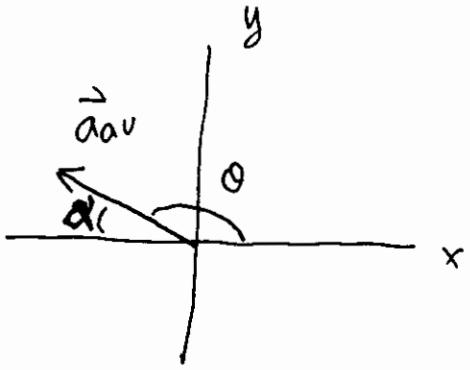
$$= \frac{1.0\cos 30^\circ - 2.5}{0.4}$$

$$= -4.08 \ m/s^2$$

$$a_{avy} = \frac{v_{By} - v_{Ay}}{\Delta t}$$

$$= \frac{1.0\sin 30^\circ - 0}{0.4}$$

$$= 1.25 \ m/s^2$$



Magnitude: $a_{av} = \sqrt{a_{avx}^2 + a_{avy}^2}$

$$= \sqrt{(-4.08)^2 + (1.25)^2}$$

$$= 4.27 \ m/s^2$$

Direction: $\alpha = \tan^{-1}\frac{|a_{avy}|}{|a_{ax}|} = 17^\circ$

OR: $\theta = \tan^{-1}\frac{a_{avy}}{a_{avx}} = 180^\circ - 17^\circ = 163^\circ$

$9_{(5/20)}$. John is driving a van on a single-lane section of the Sea-to-Sky highway. In order to pass a bus he crosses the yellow line and speeds up to 120 km/h. At the same time, a truck is coming in the opposite direction with a speed of 90 km/h. When the two vehicles are 100 m apart, both drivers hit the brakes, which results in a constant deceleration of 6.0 m/s^2 for both vehicles.

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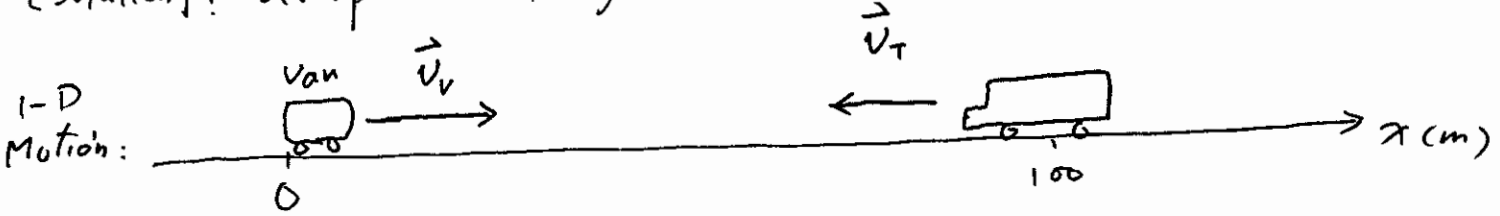

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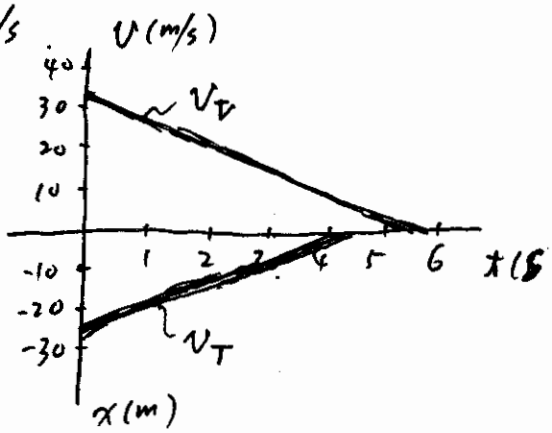

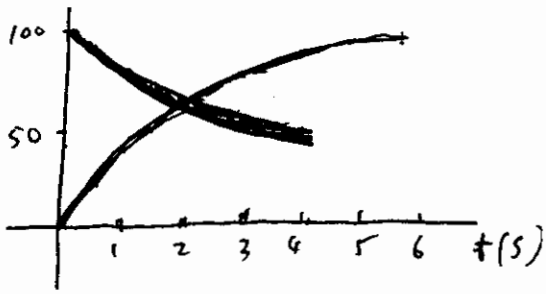

at t: $a_V = -6\ m/s^2$, $a_T = 6\ m/s^2$.

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