CHAPTER 13: Fluids

45. We may apply Torricelli's theorem, Eq. 13-9.

$$v_1 = \sqrt{2g\left(y_2 - y_1\right)} = \sqrt{2\left(9.80\,\mathrm{m/s^2}\right)\left(5.3\,\mathrm{m}\right)} = 10.2\,\mathrm{m/s} \approx \boxed{10\,\mathrm{m/s}}\ \left(2\text{ sig. fig.}\right)$$

$\boxed{49.}$ We assume that there is no appreciable height difference between the two sides of the roof. Then the net force on the roof due to the air is the difference in pressure on the two sides of the roof, times the area of the roof. The difference in pressure can be found from Bernoulli's equation.

$$P_{\text{inside}} + \tfrac{1}{2}\rho v_{\text{inside}}^2 + \rho g y_{\text{inside}} = P_{\text{outside}} + \tfrac{1}{2}\rho v_{\text{outside}}^2 + \rho g y_{\text{outside}} \quad \rightarrow$$

$$P_{\text{inside}} - P_{\text{outside}} = \tfrac{1}{2}\rho_{\text{air}} v_{\text{outside}}^2 = \frac{F_{\text{air}}}{A_{\text{roof}}} \quad \rightarrow$$

$$F_{\text{air}} = \tfrac{1}{2}\rho_{\text{air}} v_{\text{outside}}^2 A_{\text{roof}} = \tfrac{1}{2}\left(1.29\,\mathrm{kg/m^3}\right)\left[\left(180\,\mathrm{km/h}\right)\left(\frac{1\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right)\right]^2\left(6.2\,\mathrm{m}\right)\left(12.4\,\mathrm{m}\right)$$

$$= \boxed{1.2\times 10^5\,\mathrm{N}}$$

50. Use the equation of continuity (Eq. 13-7b) to relate the volume flow of water at the two locations, and use Bernoulli's equation (Eq. 13-8) to relate the pressure conditions at the two locations. We assume that the two locations are at the same height. Express the pressures as atmospheric pressure plus gauge pressure. Use subscript "1" for the larger diameter, and "2" for the smaller diameter.

$$A_1 v_1 = A_2 v_2 \quad \rightarrow \quad v_2 = v_1 \frac{A_1}{A_2} = v_1 \frac{\pi r_1^2}{\pi r_2^2} = v_1 \frac{r_1^2}{r_2^2}$$

$$P_0 + P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_0 + P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad \rightarrow$$

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2 = P_2 + \tfrac{1}{2}\rho v_1^2 \frac{r_1^4}{r_2^4} \quad \rightarrow \quad v_1 = \sqrt{\frac{2\left(P_1 - P_2\right)}{\rho\left(\frac{r_1^4}{r_2^4} - 1\right)}} \quad \rightarrow$$

$$A_1 v_1 = \pi r_1^2 \sqrt{\frac{2\left(P_1 - P_2\right)}{\rho\left(\frac{r_1^4}{r_2^4} - 1\right)}} = \pi\left(3.0\times 10^{-2}\,\mathrm{m}\right)^2 \sqrt{\frac{2\left(32.0\times 10^3\,\mathrm{Pa} - 24.0\times 10^3\,\mathrm{Pa}\right)}{\left(1.0\times 10^3\,\mathrm{kg/m^3}\right)\left(\frac{\left(3.0\times 10^{-2}\,\mathrm{m}\right)^4}{\left(2.25\times 10^{-2}\,\mathrm{m}\right)^4} - 1\right)}}$$

$$= \boxed{7.7\times 10^{-3}\,\mathrm{m^3/s}}$$

52. The lift force would be the difference in pressure between the two wing surfaces, times the area of the wing surface. The difference in pressure can be found from Bernoulli's equation. We consider the two surfaces of the wing to be at the same height above the ground. Call the bottom surface of the wing point 1, and the top surface point 2.

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad \rightarrow \quad P_1 - P_2 = \tfrac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$F_{\text{lift}} = \left(P_1 - P_2\right)\left(\text{Area of wing}\right) = \tfrac{1}{2}\rho\left(v_2^2 - v_1^2\right)A$$

$$= \tfrac{1}{2}\left(1.29\,\mathrm{kg/m^3}\right)\left[\left(280\,\mathrm{m/s}\right)^2 - \left(150\,\mathrm{m/s}\right)^2\right]\left(88\,\mathrm{m^2}\right) = \boxed{3.2\times 10^6\,\mathrm{N}}$$

54. Use the equation of continuity (Eq. 13-7b) to relate the volume flow of water at the two locations, and use Bernoulli's equation (Eq. 13-8) to relate the conditions at the street to those at the top floor. Express the pressures as atmospheric pressure plus gauge pressure.

$$A_{\text{street}} v_{\text{street}} = A_{\text{top}} v_{\text{top}} \quad \rightarrow$$

$$v_{\text{top}} = v_{\text{street}} \frac{A_{\text{street}}}{A_{\text{top}}} = (0.68\,\text{m/s}) \frac{\pi\left[\frac{1}{2}\left(5.0\times 10^{-2}\,\text{m}\right)\right]^2}{\pi\left[\frac{1}{2}\left(2.8\times 10^{-2}\,\text{m}\right)\right]^2} = 2.168\,\text{m/s} \approx \boxed{2.2\,\text{m/s}}$$

$$P_0 + P_{\substack{\text{gauge}\\ \text{street}}} + \tfrac{1}{2}\rho v_{\text{street}}^2 + \rho g y_{\text{street}} = P_0 + P_{\substack{\text{gauge}\\ \text{top}}} + \tfrac{1}{2}\rho v_{\text{top}}^2 + \rho g y_{\text{top}} \quad \rightarrow$$

$$P_{\substack{\text{gauge}\\ \text{top}}} = P_{\substack{\text{gauge}\\ \text{street}}} + \tfrac{1}{2}\rho\left(v_{\text{street}}^2 - v_{\text{top}}^2\right) + \rho g\left(y_{\text{street}} - y_{\text{top}}\right)$$

$$= (3.8\,\text{atm})\left(\frac{1.013\times 10^5\,\text{Pa}}{\text{atm}}\right) + \tfrac{1}{2}\left(1.00\times 10^3\,\text{kg/m}^3\right)\left[(0.68\,\text{m/s})^2 - (2.168\,\text{m/s})^2\right]$$

$$+ \left(1.00\times 10^3\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)(-18\,\text{m})$$

$$= 2.064\times 10^5\,\text{Pa}\left(\frac{1\,\text{atm}}{1.013\times 10^5\,\text{Pa}}\right) \approx \boxed{2.0\,\text{atm}}$$

68. Use Poiseuille's equation to find the pressure difference.

$$Q = \frac{\pi R^4 (P_2 - P_1)}{8\eta l} \quad \rightarrow$$

$$(P_2 - P_1) = \frac{8Q\eta l}{\pi R^4} = \frac{8\left(650\,\text{cm}^3/\text{s}\right)\left(10^{-6}\,\text{m}^3/\text{cm}^3\right)\left(0.20\,\text{Pa}\cdot\text{s}\right)\left(1.9\times 10^3\,\text{m}\right)}{\pi\,(0.145\,\text{m})^4}$$

$$= 1423\,\text{Pa} \approx \boxed{1400\ \text{Pa}}$$

71. The fluid pressure must be 78 torr higher than air pressure as it exits the needle, so that the blood will enter the vein. The pressure at the entrance to the needle must be higher than 78 torr, due to the viscosity of the blood. To produce that excess pressure, the blood reservoir is placed above the level of the needle. Use Poiseuille's equation to calculate the excess pressure needed due to the viscosity, and then use Eq. 13-6b to find the height of the blood reservoir necessary to produce that excess pressure.

$$Q = \frac{\pi R^4 (P_2 - P_1)}{8\eta_{\text{blood}} l} \quad \rightarrow \quad P_2 = P_1 + \frac{8\eta_{\text{blood}} l Q}{\pi R^4} = \rho_{\text{blood}} g \Delta h \quad \rightarrow$$

$$\Delta h = \frac{1}{\rho_{\text{blood}} g}\left(P_1 + \frac{8\eta_{\text{blood}} l Q}{\pi R^4}\right)$$

$$= \frac{1}{\left(1.05\times 10^3\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.80\,\text{m/s}^2\right)}\left(\begin{array}{c}(78\,\text{mm-Hg})\left(\dfrac{133\,\text{N/m}^2}{1\,\text{mm-Hg}}\right) + \\ \dfrac{8\left(4\times 10^{-3}\,\text{Pa}\cdot\text{s}\right)\left(2.5\times 10^{-2}\,\text{m}\right)\left(\dfrac{2.0\times 10^{-6}\,\text{m}^3}{60\ \text{s}}\right)}{\pi\left(0.4\times 10^{-3}\,\text{m}\right)^4}\end{array}\right)$$

$$= 1.04\,\text{m} \approx \boxed{1.0\,\text{m}}$$

Chapter 14

6. (*a*) The spring constant is found from the ratio of applied force to displacement.

$$k = \frac{F_{\text{ext}}}{x} = \frac{mg}{x} = \frac{(2.4\,\text{kg})(9.80\,\text{m/s}^2)}{0.036\,\text{m}} = 653\,\text{N/m} \approx \boxed{650\,\text{N/m}}$$

(*b*) The amplitude is the distance pulled down from equilibrium, so $A = \boxed{2.5\,\text{cm}}$

The frequency of oscillation is found from the oscillating mass and the spring constant.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{653\,\text{N/m}}{2.4\,\text{kg}}} = 2.625\,\text{Hz} \approx \boxed{2.6\,\text{Hz}}$$

12. (a) We find the effective spring constant from the mass and the frequency of oscillation.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \rightarrow$$

$$k = 4\pi^2 m f^2 = 4\pi^2 (0.055\,\text{kg})(3.0\,\text{Hz})^2 = 19.54\,\text{N/m} \approx \boxed{20\,\text{N/m}}\,(2 \text{ sig fig})$$

(b) Since the objects are the same size and shape, we anticipate that the spring constant is the same.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{19.54\,\text{N/m}}{0.25\,\text{kg}}} = \boxed{1.4\,\text{Hz}}$$

16. (*a*) From the graph, the period is 0.69 s. The period and the mass can be used to find the spring constant.

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \rightarrow \quad k = 4\pi^2\frac{m}{T^2} = 4\pi^2\frac{0.0095\,\text{kg}}{(0.69\,\text{s})^2} = 0.7877\,\text{N/m} \approx \boxed{0.79\,\text{N/m}}$$

(*b*) From the graph, the amplitude is 0.82 cm. The phase constant can be found from the initial conditions.

$$x = A\cos\left(\frac{2\pi}{T}t + \phi\right) = (0.82\,\text{cm})\cos\left(\frac{2\pi}{0.69}t + \phi\right)$$

$$x(0) = (0.82\,\text{cm})\cos\phi = 0.43\,\text{cm} \quad \rightarrow \quad \phi = \cos^{-1}\frac{0.43}{0.82} = \pm 1.02\,\text{rad}$$

Because the graph is shifted to the RIGHT from the 0-phase cosine, the phase constant must be subtracted.

$$x = \boxed{(0.82\,\text{cm})\cos\left(\frac{2\pi}{0.69}t - 1.0\right)} \text{ or } (0.82\,\text{cm})\cos(9.1t - 1.0)$$

17. (*a*) The period and frequency are found from the angular frequency.

$$\omega = 2\pi f \;\rightarrow\; f = \frac{1}{2\pi}\omega = \frac{1}{2\pi}\frac{5\pi}{4} = \boxed{\frac{5}{8}\,\mathrm{Hz}} \qquad T = \frac{1}{f} = \boxed{1.6\ \mathrm{s}}$$

(*b*) The velocity is the derivative of the position.

$$x = (3.8\,\mathrm{m})\cos\left(\frac{5\pi}{4}t + \frac{\pi}{6}\right) \qquad v = \frac{dx}{dt} = -(3.8\,\mathrm{m})\left(\frac{5\pi}{4}\right)\sin\left(\frac{5\pi}{4}t + \frac{\pi}{6}\right)$$

$$x(0) = (3.8\,\mathrm{m})\cos\left(\frac{\pi}{6}\right) = \boxed{3.3\,\mathrm{m}} \qquad v(0) = -(3.8\,\mathrm{m})\left(\frac{5\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) = \boxed{-7.5\,\mathrm{m/s}}$$

(*c*) The acceleration is the derivative of the velocity.

$$v = -(3.8\,\mathrm{m})\left(\frac{5\pi}{4}\right)\sin\left(\frac{5\pi}{4}t + \frac{\pi}{6}\right) \qquad a = \frac{dv}{dt} = -(3.8\,\mathrm{m})\left(\frac{5\pi}{4}\right)^2\cos\left(\frac{5\pi}{4}t + \frac{\pi}{6}\right)$$

$$v(2.0) = -(3.8\,\mathrm{m})\left(\frac{5\pi}{4}\right)\sin\left(\frac{5\pi}{4}(2.0) + \frac{\pi}{6}\right) = \boxed{-13\,\mathrm{m/s}}$$

$$a(2.0) = -(3.8\,\mathrm{m})\left(\frac{5\pi}{4}\right)^2\cos\left(\frac{5\pi}{4}(2.0) + \frac{\pi}{6}\right) = \boxed{29\,\mathrm{m/s^2}}$$

30. (*a*) At equilibrium, the velocity is its maximum. Use Eq. 14-9a, and realize that the object can be moving in either direction.

$$v_{\mathrm{max}} = \omega A = 2\pi f A = 2\pi(2.5\,\mathrm{Hz})(0.15\,\mathrm{m}) = 2.356\,\mathrm{m/s} \;\rightarrow\; v_{\mathrm{equib}} \approx \boxed{\pm 2.4\,\mathrm{m/s}}$$

(*b*) From Eq. 14-11b, we find the velocity at any position.

$$v = \pm v_{\mathrm{max}}\sqrt{1 - \frac{x^2}{A^2}} = \pm(2.356\,\mathrm{m/s})\sqrt{1 - \frac{(0.10\,\mathrm{m})^2}{(0.15\,\mathrm{m})^2}} = \pm 1.756\,\mathrm{m/s} \approx \boxed{\pm 1.8\,\mathrm{m/s}}$$

(*c*) $E_{\mathrm{total}} = \frac{1}{2}mv_{\mathrm{max}}^2 = \frac{1}{2}(0.35\,\mathrm{kg})(2.356\,\mathrm{m/s})^2 = 0.9714\,\mathrm{J} \approx \boxed{0.97\,\mathrm{J}}$

(*d*) Since the object has a maximum displacement at $t = 0$, the position will be described by the cosine function.

$$x = (0.15\,\mathrm{m})\cos\left(2\pi(2.5\ \mathrm{Hz})t\right) \;\rightarrow\; \boxed{x = (0.15\,\mathrm{m})\cos(5.0\pi\, t)}$$

$\boxed{37.}$ We assume that the collision of the bullet and block is so quick that there is no significant motion of the large mass or spring during the collision. Linear momentum is conserved in this collision. The speed that the combination has right after the collision is the maximum speed of the oscillating system. Then, the kinetic energy that the combination has right after the collision is stored in the spring when it is fully compressed, at the amplitude of its motion.

$$p_{\mathrm{before}} = p_{\mathrm{after}} \;\rightarrow\; mv_0 = (m+M)v_{\mathrm{max}} \;\rightarrow\; v_{\mathrm{max}} = \frac{m}{m+M}v_0$$

$$\tfrac{1}{2}(m+M)v_{\mathrm{max}}^2 = \tfrac{1}{2}kA^2 \;\rightarrow\; \tfrac{1}{2}(m+M)\left(\frac{m}{m+M}v_0\right)^2 = \tfrac{1}{2}kA^2 \;\rightarrow$$

$$v_0 = \frac{A}{m}\sqrt{k(m+M)} = \frac{(9.460\times10^{-2}\,\mathrm{m})}{(7.870\times10^{-3}\,\mathrm{kg})}\sqrt{(142.7\,\mathrm{N/m})(7.870\times10^{-3}\,\mathrm{kg} + 4.648\,\mathrm{kg})}$$

$$= \boxed{309.8\,\mathrm{m/s}}$$

41. The period of a pendulum is given by $T = 2\pi\sqrt{L/g}$. The length is assumed to be the same for the pendulum both on Mars and on Earth.

$$T = 2\pi\sqrt{L/g} \quad \rightarrow \quad \frac{T_{\text{Mars}}}{T_{\text{Earth}}} = \frac{2\pi\sqrt{L/g_{\text{Mars}}}}{2\pi\sqrt{L/g_{\text{Earth}}}} = \sqrt{\frac{g_{\text{Earth}}}{g_{\text{Mars}}}} \quad \rightarrow$$

$$T_{\text{Mars}} = T_{\text{Earth}}\sqrt{\frac{g_{\text{Earth}}}{g_{\text{Mars}}}} = (1.35\,\text{s})\sqrt{\frac{1}{0.37}} = \boxed{2.2\,\text{s}}$$

46. There are $(24\,\text{h})(60\,\text{min/h})(60\,\text{s/min}) = 86,400\,\text{s}$ in a day. The clock should make one cycle in exactly two seconds (a "tick" and a "tock"), and so the clock should make 43,200 cycles per day. After one day, the clock in question is 26 seconds slow, which means that it has made 13 less cycles than required for precise timekeeping. Thus the clock is only making 43,187 cycles in a day. Accordingly, the period of the clock must be decreased by a factor of $\frac{43,187}{43,200}$.

$$T_{\text{new}} = \frac{43,187}{43,200}T_{\text{old}} \quad \rightarrow \quad 2\pi\sqrt{l_{\text{new}}/g} = \left(\frac{43,187}{43,200}\right)2\pi\sqrt{l_{\text{old}}/g} \quad \rightarrow$$

$$l_{\text{new}} = \left(\frac{43,187}{43,200}\right)^2 l_{\text{old}} = \left(\frac{43,187}{43,200}\right)^2 (0.9930\,\text{m}) = 0.9924\,\text{m}$$

Thus the pendulum should be $\boxed{\text{shortened by 0.6 mm}}$.