

46. During the final part of the race, the runner must have a displacement of 1100 m in a time of 180 s (3.0 min). Assume that the starting speed for the final part is the same as the average speed thus far.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{8900 \text{ m}}{(27 \times 60) \text{ s}} = 5.494 \text{ m/s} = v_0$$

The runner will accomplish this by accelerating from speed  $v_0$  to speed  $v$  for  $t$  seconds, covering a distance  $d_1$ , and then running at a constant speed of  $v$  for  $(180 - t)$  seconds, covering a distance  $d_2$ . We have these relationships from Eq. 2-12a and Eq. 2-12b.

$$v = v_0 + at \quad d_1 = v_0 t + \frac{1}{2} at^2 \quad d_2 = v(180 - t) = (v_0 + at)(180 - t)$$

$$1100 \text{ m} = d_1 + d_2 = v_0 t + \frac{1}{2} at^2 + (v_0 + at)(180 - t) \rightarrow 1100 \text{ m} = 180v_0 + 180at - \frac{1}{2} at^2 \rightarrow$$

$$1100 \text{ m} = (180 \text{ s})(5.494 \text{ m/s}) + (180 \text{ s})(0.2 \text{ m/s}^2)t - \frac{1}{2}(0.2 \text{ m/s}^2)t^2$$

$$0.1t^2 - 36t + 111 = 0 \quad t = 357 \text{ s}, 3.11 \text{ s}$$

Since we must have  $t < 180 \text{ s}$ , the solution is  $t = 3.1 \text{ s}$ .

47. For the runners to cross the finish line side-by-side means they must both reach the finish line in the same amount of time from their current positions. Take Mary's current location as the origin. Use Eq. 2-12b.

$$\text{For Sally:} \quad 22 = 5 + 5t + \frac{1}{2}(-.5)t^2 \rightarrow t^2 - 20t + 68 = 0 \rightarrow$$

$$t = \frac{20 \pm \sqrt{20^2 - 4(68)}}{2} = 4.343 \text{ s}, 15.66 \text{ s}$$

The first time is the time she first crosses the finish line, and so is the time to be used for the problem. Now find Mary's acceleration so that she crosses the finish line in that same amount of time.

$$\text{For Mary:} \quad 22 = 0 + 4t + \frac{1}{2} at^2 \rightarrow a = \frac{22 - 4t}{\frac{1}{2} t^2} = \frac{22 - 4(4.343)}{\frac{1}{2}(4.343)^2} = 0.49 \text{ m/s}^2$$