CHAPTER 2: Describing Motion: Kinematics in One Dimension

62. Choose upward to be the positive direction, and $y_0 = 0$ to be the location of the nozzle. The initial velocity is v_0, the acceleration is $a = -9.80\,\text{m/s}^2$, the final location is $y = -1.5\,\text{m}$, and the time of flight is $t = 2.0\,\text{s}$. Using Eq. 2-12b and substituting *y* for *x* gives the following.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad v_0 = \frac{y - \frac{1}{2}at^2}{t} = \frac{-1.5\text{ m} - \frac{1}{2}\left(-9.80\,\text{m/s}^2\right)\left(2.0\text{ s}\right)^2}{2.0\text{ s}} = \boxed{9.1\,\text{m/s}}$$

$\boxed{67.}$ The displacement is found from the integral of the velocity, over the given time interval.

$$\Delta x = \int_{t_1}^{t_2} v\,dt = \int_{t=1.5\text{s}}^{t=3.1\text{s}} \left(25 + 18t\right)dt = \left(25t + 9t^2\right)\Big|_{t=1.5\text{s}}^{t=3.1\text{s}} = \left[25(3.1) + 9(3.1)^2\right] - \left[25(1.5) + 9(1.5)^2\right]$$

$$= \boxed{106\,\text{m}}$$

The same result can be obtained by evaluating the area under the v-t line.

81. Choose downward to be the positive direction, and $y_0 = 0$ to be at the top of the cliff. The initial velocity is $v_0 = -12.5\,\text{m/s}$, the acceleration is $a = 9.80\,\text{m/s}^2$, and the final location is $y = 75.0\,\text{m}$.

(*a*) Using Eq. 2-12b and substituting *y* for *x*, we have the following.

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \rightarrow \left(4.9\,\text{m/s}^2\right)t^2 - \left(12.5\,\text{m/s}\right)t - 75.0\text{ m} = 0 \rightarrow t = -2.839\,\text{s}\,,\ 5.390\,\text{s}$$

The positive answer is the physical answer: $\boxed{t = 5.39\,\text{s}}$.

(*b*) Using Eq. 2-12a, we have $v = v_0 + at = -12.5\,\text{m/s} + \left(9.80\,\text{m/s}^2\right)\left(5.390\,\text{s}\right) = \boxed{40.3\,\text{m/s}}$.

(*c*) The total distance traveled will be the distance up plus the distance down. The distance down will be 75.0 m more than the distance up. To find the distance up, use the fact that the speed at the top of the path will be 0. Using Eq. 2-12c we have the following.

$$v^2 = v_0^2 + 2a\left(y - y_0\right) \quad \rightarrow \quad y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - \left(-12.5\,\text{m/s}\right)^2}{2\left(9.80\,\text{m/s}^2\right)} = -7.97\,\text{m}$$

Thus the distance up is 7.97 m, the distance down is 82.97 m, and the total distance traveled is $\boxed{90.9\,\text{m}}$.

CHAPTER 3: Kinematics in Two or Three Dimensions; Vectors

7. (*a*) $v_{\text{north}} = (835\,\text{km/h})(\cos 41.5°) = \boxed{625\,\text{km/h}}$ $\quad v_{\text{west}} = (835\,\text{km/h})(\sin 41.5°) = \boxed{553\,\text{km/h}}$

(*b*) $\Delta d_{\text{north}} = v_{\text{north}} t = (625\ \text{km/h})(2.50\ \text{h}) = \boxed{1560\ \text{km}}$

$\Delta d_{\text{west}} = v_{\text{west}} t = (553\ \text{km/h})(2.50\ \text{h}) = \boxed{1380\ \text{km}}$

13. $A_x = 44.0\cos 28.0° = 38.85$ $\quad A_y = 44.0\sin 28.0° = 20.66$

$B_x = -26.5\cos 56.0° = -14.82$ $\quad B_y = 26.5\sin 56.0° = 21.97$

$C_x = 31.0\cos 270° = 0.0$ $\quad C_y = 31.0\sin 270° = -31.0$

(*a*) $(\vec{\mathbf{B}} - 2\vec{\mathbf{A}})_x = -14.82 - 2(38.85) = -92.52$ $\quad (\vec{\mathbf{B}} - 2\vec{\mathbf{A}})_y = 21.97 - 2(20.66) = -19.35$

Note that since both components are negative, the vector is in the 3rd quadrant.

$\vec{\mathbf{B}} - 2\vec{\mathbf{A}} = \boxed{-92.5\hat{\mathbf{i}} - 19.4\hat{\mathbf{j}}}$

$\left|\vec{\mathbf{B}} - 2\vec{\mathbf{A}}\right| = \sqrt{(-92.52)^2 + (-19.35)^2} = \boxed{94.5}$ $\quad \theta = \tan^{-1}\dfrac{-19.35}{-92.52} = \boxed{11.8°\text{below} - x \text{ axis}}$

(*b*) $(2\vec{\mathbf{A}} - 3\vec{\mathbf{B}} + 2\vec{\mathbf{C}})_x = 2(38.85) - 3(-14.82) + 2(0.0) = 122.16$

$(2\vec{\mathbf{A}} - 3\vec{\mathbf{B}} + 2\vec{\mathbf{C}})_y = 2(20.66) - 3(21.97) + 2(-31.0) = -86.59$

Note that since the x component is positive and the y component is negative, the vector is in the 4th quadrant.

$2\vec{\mathbf{A}} - 3\vec{\mathbf{B}} + 2\vec{\mathbf{C}} = \boxed{122\hat{\mathbf{i}} - 86.6\hat{\mathbf{j}}}$

$\left|2\vec{\mathbf{A}} - 3\vec{\mathbf{B}} + 2\vec{\mathbf{C}}\right| = \sqrt{(122.16)^2 + (-86.59)^2} = \boxed{150}$ $\quad \theta = \tan^{-1}\dfrac{-86.59}{122.16} = \boxed{35.3°\text{below} + x \text{ axis}}$

25. (*a*) Differentiate the position vector, $\vec{\mathbf{r}} = (3.0t^2\,\hat{\mathbf{i}} - 6.0t^3\hat{\mathbf{j}})\,\text{m}$, with respect to time in order to find the velocity and the acceleration.

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \boxed{(6.0\,t\,\hat{\mathbf{i}} - 18.0t^2\hat{\mathbf{j}})\,\text{m/s}} \qquad \vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \boxed{(6.0\,\hat{\mathbf{i}} - 36.0t\,\hat{\mathbf{j}})\,\text{m/s}^2}$$

(*b*) $\vec{\mathbf{r}}(2.5\,\text{s}) = \left[3.0(2.5)^2\,\hat{\mathbf{i}} - 6.0(2.5)^3\,\hat{\mathbf{j}}\right]\text{m} = \boxed{(19\,\hat{\mathbf{i}} - 94\hat{\mathbf{j}})\,\text{m}}$

$\vec{\mathbf{v}}(2.5\,\text{s}) = \left[6.0(2.5)\,\hat{\mathbf{i}} - 18.0(2.5)^2\,\hat{\mathbf{j}}\right]\text{m/s} = \boxed{(15\,\hat{\mathbf{i}} - 110\hat{\mathbf{j}})\,\text{m/s}}$

41. (*a*) Take the ground to be the $y = 0$ level, with upward as the positive direction. Use Eq. 2-12b to solve for the time, with an initial vertical velocity of 0.

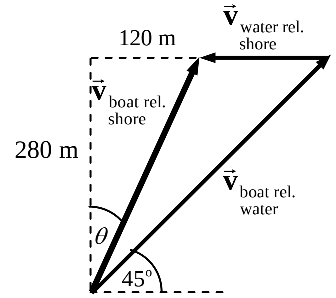
$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 150\,\text{m} = 910\,\text{m} + \tfrac{1}{2}(-9.80\,\text{m/s}^2)t^2 \quad \rightarrow$$

$$t = \sqrt{\frac{2(150 - 910)}{(-9.80\,\text{m/s}^2)}} = 12.45\,\text{s} \approx \boxed{12\,\text{s}}$$

(*b*) The horizontal motion is at a constant speed, since air resistance is being ignored.

$$\Delta x = v_x t = (5.0\,\text{m/s})(12.45\,\text{s}) = 62.25\,\text{m} \approx \boxed{62\,\text{m}}$$

70. Call the direction of the flow of the river the x direction (to the left in the diagram), and the direction straight across the river the y direction (to the top in the diagram). From the diagram, $\theta = \tan^{-1} 120\,\text{m}/280\,\text{m} = 23°$. Equate the vertical components of the velocities to find the speed of the boat relative to the shore.



$$v_{\substack{\text{boat rel.}\\\text{shore}}} \cos\theta = v_{\substack{\text{boat rel.}\\\text{water}}} \sin 45° \quad \rightarrow$$

$$v_{\substack{\text{boat rel.}\\\text{shore}}} = (2.70\,\text{m/s}) \frac{\sin 45°}{\cos 23°} = 2.07\,\text{m/s}$$

Equate the horizontal components of the velocities.

$$v_{\substack{\text{boat rel.}\\\text{shore}}} \sin\theta = v_{\substack{\text{boat rel.}\\\text{water}}} \cos 45° - v_{\substack{\text{water}\\\text{rel. shore}}} \quad \rightarrow$$

$$v_{\substack{\text{water}\\\text{rel. shore}}} = v_{\substack{\text{boat rel.}\\\text{water}}} \cos 45° - v_{\substack{\text{boat rel.}\\\text{shore}}} \sin\theta$$

$$= (2.70\,\text{m/s})\cos 45° - (2.07\,\text{m/s})\sin 23° = \boxed{1.10\,\text{m/s}}$$

77. Choose upward to be the positive y direction. The origin is the point from which the pebbles are released. In the vertical direction, $a_y = -9.80\,\text{m/s}^2$, the velocity at the window is $v_y = 0$, and the vertical displacement is 8.0 m. The initial y velocity is found from Eq. 2-12c.

$$v_y^2 = v_{y0}^2 + 2a_y(y - y_0) \quad \rightarrow$$

$$v_{y0} = \sqrt{v_y^2 - 2a_y(y - y_0)} = \sqrt{0 - 2(-9.80\,\text{m/s}^2)(8.0\ \text{m})} = 12.5\,\text{m/s}$$

Find the time for the pebbles to travel to the window from Eq. 2-12a.

$$v_y = v_{y0} + at \quad \rightarrow \quad t = \frac{v_y - v_{y0}}{a} = \frac{0 - 12.5\,\text{m/s}}{-9.80\,\text{m/s}^2} = 1.28\,\text{s}$$

Find the horizontal speed from the horizontal motion at constant velocity.

$$\Delta x = v_x t \quad \rightarrow \quad v_x = \Delta x/t = 9.0\ \text{m}/1.28\ \text{s} = \boxed{7.0\,\text{m/s}}$$

This is the speed of the pebbles when they hit the window.