CHAPTER 3: Kinematics in Two or Three Dimensions; Vectors

46. Choose the origin to be at ground level, under the place where the projectile is launched, and upwards to be the positive *y* direction. For the projectile, $v_0 = 65.0\,\mathrm{m/s}$, $\theta_0 = 35.0°$, $a_y = -g$, $y_0 = 115\,\mathrm{m}$, and $v_{y0} = v_0 \sin\theta_0$.

(*a*) The time taken to reach the ground is found from Eq. 2-12b, with a final height of 0.

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad 0 = y_0 + v_0 \sin\theta_0 t - \tfrac{1}{2}gt^2 \quad \rightarrow$$

$$t = \frac{-v_0\sin\theta_0 \pm \sqrt{v_0^2 \sin^2\theta_0 - 4\left(-\tfrac{1}{2}g\right)y_0}}{2\left(-\tfrac{1}{2}g\right)} = 9.964\,\mathrm{s}\ ,\ -2.3655\,\mathrm{s} = \boxed{9.96\,\mathrm{s}}$$

Choose the positive time since the projectile was launched at time $t = 0$.

(*b*) The horizontal range is found from the horizontal motion at constant velocity.

$$\Delta x = v_x t = \left(v_0 \cos\theta_0\right)t = \left(65.0\,\mathrm{m/s}\right)\left(\cos 35.0°\right)\left(9.964\,\mathrm{s}\right) = \boxed{531\,\mathrm{m}}$$

(*c*) At the instant just before the particle reaches the ground, the horizontal component of its velocity is the constant $v_x = v_0\cos\theta_0 = \left(65.0\,\mathrm{m/s}\right)\cos 35.0° = \boxed{53.2\,\mathrm{m/s}}$. The vertical component is found from Eq. 2-12a.

$$v_y = v_{y0} + at = v_0\sin\theta_0 - gt = \left(65.0\,\mathrm{m/s}\right)\sin 35.0° - \left(9.80\,\mathrm{m/s^2}\right)\left(9.964\,\mathrm{s}\right)$$

$$= \boxed{-60.4\,\mathrm{m/s}}$$

(*d*) The magnitude of the velocity is found from the *x* and *y* components calculated in part (*c*) above.

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(53.2\,\mathrm{m/s}\right)^2 + \left(-60.4\,\mathrm{m/s}\right)^2} = \boxed{80.5\,\mathrm{m/s}}$$

(*e*) The direction of the velocity is $\theta = \tan^{-1}\dfrac{v_y}{v_x} = \tan^{-1}\dfrac{-60.4}{53.2} = -48.6°$, and so the object is moving $\boxed{48.6°\text{ below the horizon}}$.

(*f*) The maximum height above the cliff top reached by the projectile will occur when the *y*-velocity is 0, and is found from Eq. 2-12c.

$$v_y^2 = v_{y0}^2 + 2a_y\left(y - y_0\right) \quad \rightarrow \quad 0 = v_0^2\sin^2\theta_0 - 2gy_{\max}$$

$$y_{\max} = \frac{v_0^2\sin^2\theta_0}{2g} = \frac{\left(65.0\,\mathrm{m/s}\right)^2\sin^2 35.0°}{2\left(9.80\,\mathrm{m/s^2}\right)} = \boxed{70.9\,\mathrm{m}}$$

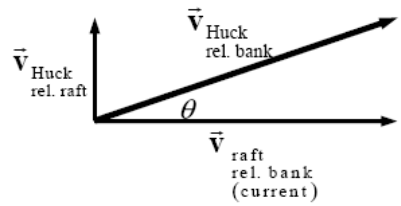
57. Call the direction of the boat relative to the water the positive direction. For the jogger moving towards the bow, we have the following:

$$\vec{\mathbf{v}}_{\substack{\text{jogger}\\ \text{rel. water}}} = \vec{\mathbf{v}}_{\substack{\text{jogger}\\ \text{rel. boat}}} + \vec{\mathbf{v}}_{\substack{\text{boat rel.}\\ \text{water}}} = 2.0\,\text{m/s}\,\hat{\mathbf{i}} + 8.5\,\text{m/s}\,\hat{\mathbf{i}} = \boxed{10.5\,\text{m/s}\,\hat{\mathbf{i}}}.$$

For the jogger moving towards the stern, we have the following.

$$\vec{\mathbf{v}}_{\substack{\text{jogger}\\ \text{rel. water}}} = \vec{\mathbf{v}}_{\substack{\text{jogger}\\ \text{rel. boat}}} + \vec{\mathbf{v}}_{\substack{\text{boat rel.}\\ \text{water}}} = -2.0\,\text{m/s}\,\hat{\mathbf{i}} + 8.5\,\text{m/s}\,\hat{\mathbf{i}} = \boxed{6.5\,\text{m/s}\,\hat{\mathbf{i}}}$$

58. Call the direction of the flow of the river the x direction, and the direction of Huck walking relative to the raft the y direction.



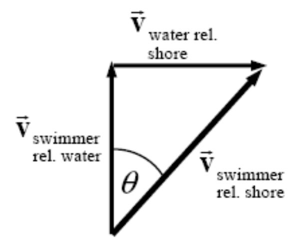
$$\vec{\mathbf{v}}_{\substack{\text{Huck}\\ \text{rel. bank}}} = \vec{\mathbf{v}}_{\substack{\text{Huck}\\ \text{rel. raft}}} + \vec{\mathbf{v}}_{\substack{\text{raft rel.}\\ \text{bank}}} = 0.70\hat{\mathbf{j}}\,\text{m/s} + 1.50\hat{\mathbf{i}}\,\text{m/s}$$

$$= \left(1.50\hat{\mathbf{i}} + 0.70\hat{\mathbf{j}}\right)\text{m/s}$$

Magnitude: $v_{\substack{\text{Huck}\\ \text{rel. bank}}} = \sqrt{1.50^2 + 0.70^2} = \boxed{1.66\,\text{m/s}}$

Direction: $\theta = \tan^{-1}\dfrac{0.70}{1.50} = \boxed{25^\circ \text{relative to river}}$

$\boxed{67.}$ Call the direction of the flow of the river the x direction, and the direction straight across the river the y direction. Call the location of the swimmer's starting point the origin.



$$\vec{\mathbf{v}}_{\substack{\text{swimmer}\\ \text{rel. shore}}} = \vec{\mathbf{v}}_{\substack{\text{swimmer}\\ \text{rel. water}}} + \vec{\mathbf{v}}_{\substack{\text{water rel.}\\ \text{shore}}} = 0.60\,\text{m/s}\,\hat{\mathbf{j}} + 0.50\,\text{m/s}\,\hat{\mathbf{i}}$$

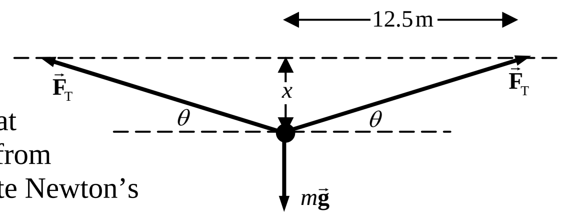
(*a*) Since the swimmer starts from the origin, the distances covered in the x and y directions will be exactly proportional to the speeds in those directions.

$$\frac{\Delta x}{\Delta y} = \frac{v_x t}{v_y t} = \frac{v_x}{v_y} \quad\rightarrow\quad \frac{\Delta x}{55\text{ m}} = \frac{0.50\,\text{m/s}}{0.60\,\text{m/s}} \quad\rightarrow\quad \Delta x = \boxed{46\,\text{m}}$$

(*b*) The time is found from the constant velocity relationship for either the x or y directions.

$$\Delta y = v_y t \quad\rightarrow\quad t = \frac{\Delta y}{v_y} = \frac{55\text{ m}}{0.60\,\text{m/s}} = \boxed{92\,\text{s}}$$

$\boxed{31.}$ (*a*) We draw a free-body diagram for the piece of the rope that is directly above the person. That piece of rope should be in equilibrium. The person's weight will be pulling down on that spot, and the rope tension will be pulling away from that spot towards the points of attachment. Write Newton's second law for that small piece of the rope.



$$\sum F_y = 2F_T \sin\theta - mg = 0 \quad \rightarrow \quad \theta = \sin^{-1}\frac{mg}{2F_T} = \sin^{-1}\frac{(72.0\,\text{kg})(9.80\,\text{m/s}^2)}{2(2900\,\text{N})} = 6.988^\circ$$

$$\tan\theta = \frac{x}{12.5\,\text{m}} \quad \rightarrow \quad x = (12.5\,\text{m})\tan 6.988^\circ = 1.532\,\text{m} \approx \boxed{1.5\,\text{m}}$$

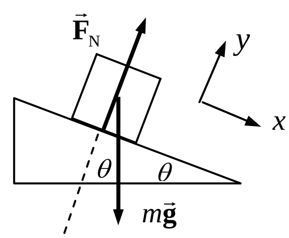
(*b*) Use the same equation to solve for the tension force with a sag of only ¼ that found above.

$$x = \tfrac{1}{4}(1.532\,\text{m}) = 0.383\,\text{m} \;;\; \theta = \tan^{-1}\frac{0.383\,\text{m}}{12.5\,\text{m}} = 1.755^\circ$$

$$F_T = \frac{mg}{2\sin\theta} = \frac{(72.0\,\text{kg})(9.80\,\text{m/s}^2)}{2(\sin 1.755^\circ)} = \boxed{11.5\,\text{kN}}$$

The $\boxed{\text{rope will not break}}$, but it exceeds the recommended tension by a factor of about 4.

48. (*a*) Consider the free-body diagram for the block on the frictionless surface. There is no acceleration in the *y* direction. Use Newton's second law for the *x* direction to find the acceleration.



$$\sum F_x = mg\sin\theta = ma \quad \rightarrow$$

$$a = g\sin\theta = (9.80\,\text{m/s}^2)\sin 22.0^\circ = \boxed{3.67\,\text{m/s}^2}$$

(*b*) Use Eq. 2-12c with $v_0 = 0$ to find the final speed.

$$v^2 - v_0^2 = 2a(x - x_0) \quad \rightarrow \quad v = \sqrt{2a(x-x_0)} = \sqrt{2(3.67\,\text{m/s}^2)(12.0\,\text{m})} = \boxed{9.39\,\text{m/s}}$$

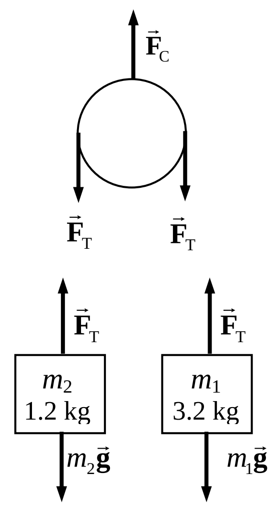
53. This problem can be solved in the same way as problem 51, with the modification that we increase mass m_A by the mass of l_A and we increase mass m_B by the mass of l_B. We take the result from problem 51 for the acceleration and make these modifications. We assume that the cord is uniform, and so the mass of any segment is directly proportional to the length of that segment.

$$a = g\frac{m_B}{m_A + m_B} \quad \rightarrow \quad a = g\frac{m_B + \dfrac{l_B}{l_A + l_B}m_C}{\left(m_A + \dfrac{l_A}{l_A + l_B}m_C\right) + \left(m_B + \dfrac{l_B}{l_A + l_B}m_C\right)} = \boxed{g\frac{m_B + \dfrac{l_B}{l_A + l_B}m_C}{m_A + m_B + m_C}}$$

Note that this acceleration is NOT constant, because the lengths l_A and l_B are functions of time. Thus constant acceleration kinematics would not apply to this system.

54. We draw a free-body diagram for each mass. We choose UP to be the positive direction. The tension force in the cord is found from analyzing the two hanging masses. Notice that the same tension force is applied to each mass. Write Newton's second law for each of the masses.

$$F_T - m_1 g = m_1 a_1 \qquad F_T - m_2 g = m_2 a_2$$



Since the masses are joined together by the cord, their accelerations will have the same magnitude but opposite directions. Thus $a_1 = -a_2$.

Substitute this into the force expressions and solve for the tension force.

$$F_T - m_1 g = -m_1 a_2 \quad \rightarrow \quad F_T = m_1 g - m_1 a_2 \quad \rightarrow \quad a_2 = \frac{m_1 g - F_T}{m_1}$$

$$F_T - m_2 g = m_2 a_2 = m_2 \left(\frac{m_1 g - F_T}{m_1} \right) \quad \rightarrow \quad F_T = \frac{2 m_1 m_2 g}{m_1 + m_2}$$

Apply Newton's second law to the stationary pulley.

$$F_C - 2F_T = 0 \quad \rightarrow \quad F_C = 2F_T = \frac{4 m_1 m_2 g}{m_1 + m_2} = \frac{4(3.2\,\text{kg})(1.2\,\text{kg})(9.80\,\text{m/s}^2)}{4.4\,\text{kg}} = \boxed{34\,\text{N}}$$

35. Choose the *y* direction to be the "forward" direction for the motion of the snowcats, and the *x* direction to be to the right on the diagram in the textbook. Since the housing unit moves in the forward direction on a straight line, there is no acceleration in the *x* direction, and so the net force in the *x* direction must be 0. Write Newton's second law for the *x* direction.

$$\sum F_x = F_{Ax} + F_{Bx} = 0 \quad \rightarrow \quad -F_A \sin 48^\circ + F_B \sin 32^\circ = 0 \quad \rightarrow$$

$$F_B = \frac{F_A \sin 48^\circ}{\sin 32^\circ} = \frac{(4500\,\text{N}) \sin 48^\circ}{\sin 32^\circ} = 6311\,\text{N} \approx \boxed{6300\,\text{N}}$$

Since the *x* components add to 0, the magnitude of the vector sum of the two forces will just be the sum of their *y* components.

$$\sum F_y = F_{Ay} + F_{By} = F_A \cos 48^\circ + F_B \cos 32^\circ = (4500\,\text{N}) \cos 48^\circ + (6311\,\text{N}) \cos 32^\circ$$

$$= 8363\,\text{N} \approx \boxed{8400\,\text{N}}$$