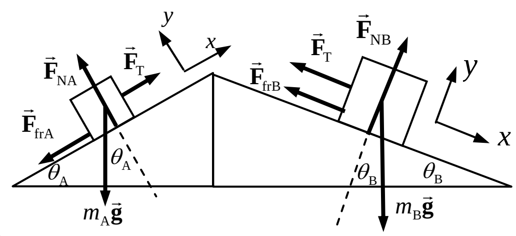
CHAPTER 5

28. We define the positive x direction to be the direction of motion for each block. See the free-body diagrams. Write Newton's second law in both dimensions for both objects. Add the two x-equations to find the acceleration.



Block A:

$$\sum F_{y\mathrm{A}} = F_{\mathrm{NA}} - m_{\mathrm{A}} g\cos\theta_{\mathrm{A}} = 0 \;\rightarrow\; F_{\mathrm{NA}} = m_{\mathrm{A}} g\cos\theta_{\mathrm{A}}$$

$$\sum F_{x\mathrm{A}} = F_{\mathrm{T}} - m_{\mathrm{A}} g\sin\theta - F_{\mathrm{frA}} = m_{\mathrm{A}} a$$

Block B:

$$\sum F_{yB} = F_{\mathrm{NB}} - m_{\mathrm{B}} g\cos\theta_{\mathrm{B}} = 0 \;\rightarrow\; F_{\mathrm{NB}} = m_{\mathrm{B}} g\cos\theta_{\mathrm{B}}$$

$$\sum F_{xB} = m_{\mathrm{B}} g\sin\theta - F_{\mathrm{frB}} - F_{\mathrm{T}} = m_{\mathrm{B}} a$$

Add the final equations together from both analyses and solve for the acceleration, noting that in both cases the friction force is found as $F_{\mathrm{fr}} = \mu F_{\mathrm{N}}$.

$$m_{\mathrm{A}} a = F_{\mathrm{T}} - m_{\mathrm{A}} g\sin\theta_{\mathrm{A}} - \mu_{\mathrm{A}} m_{\mathrm{A}} g\cos\theta_{\mathrm{A}} \quad ; \quad m_{\mathrm{B}} a = m_{\mathrm{B}} g\sin\theta_{\mathrm{B}} - \mu_{\mathrm{B}} m_{\mathrm{B}} g\cos\theta_{\mathrm{B}} - F_{\mathrm{T}}$$

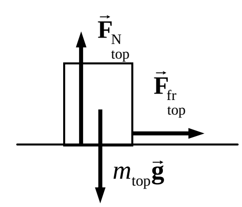
$$m_{\mathrm{A}} a + m_{\mathrm{B}} a = F_{\mathrm{T}} - m_{\mathrm{A}} g\sin\theta_{\mathrm{A}} - \mu_{\mathrm{A}} m_{\mathrm{A}} g\cos\theta_{\mathrm{A}} + m_{\mathrm{B}} g\sin\theta_{\mathrm{B}} - \mu_{\mathrm{B}} m_{\mathrm{B}} g\cos\theta_{\mathrm{B}} - F_{\mathrm{T}} \quad \rightarrow$$

$$a = g\left[\frac{-m_{\mathrm{A}}\left(\sin\theta_{\mathrm{A}} + \mu_{\mathrm{A}}\cos\theta_{\mathrm{A}}\right) + m_{\mathrm{B}}\left(\sin\theta - \mu_{\mathrm{B}}\cos\theta\right)}{\left(m_{\mathrm{A}} + m_{\mathrm{B}}\right)}\right]$$

$$= \left(9.80\,\mathrm{m/s^2}\right)\left[\frac{-(2.0\,\mathrm{kg})(\sin 51^\circ + 0.30\cos 51^\circ) + (5.0\,\mathrm{kg})(\sin 21^\circ - 0.30\cos 21^\circ)}{(7.0\,\mathrm{kg})}\right]$$

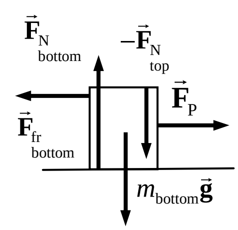
$$= \boxed{-2.2\,\mathrm{m/s^2}}$$

32. Free-body diagrams are shown for both blocks. There is a force of friction between the two blocks, which acts to the right on the top block, and to the left on the bottom block. They are a Newton's third law pair of forces.



(*a*) If the 4.0 kg block does not slide off, then it must have the same acceleration as the 12.0 kg block. That acceleration is caused by the force of static friction between the two blocks. To find the minimum coefficient, we use the maximum force of static friction.

$$F_{\substack{\mathrm{fr}\\ \mathrm{top}}} = m_{\mathrm{top}} a = \mu F_{\substack{\mathrm{N}\\ \mathrm{top}}} = \mu m_{\mathrm{top}} g \;\rightarrow\; \mu = \frac{a}{g} = \frac{5.2\,\mathrm{m/s^2}}{9.80\,\mathrm{m/s^2}} = 0.5306 \approx \boxed{0.53}$$



(*b*) If the coefficient of friction only has half the value, then the blocks will be sliding with respect to one another, and so the friction will be kinetic.

$$\mu = \tfrac{1}{2}(0.5306) = 0.2653 \;;\; F_{\substack{\mathrm{fr}\\ \mathrm{top}}} = m_{\mathrm{top}} a = \mu F_{\substack{\mathrm{N}\\ \mathrm{top}}} = \mu m_{\mathrm{top}} g \quad \rightarrow$$

$$a = \mu g = (0.2653)\left(9.80\,\mathrm{m/s^2}\right) = \boxed{2.6\,\mathrm{m/s^2}}$$

(*c*) The bottom block is still accelerating to the right at $5.2\,\mathrm{m/s^2}$. Since the top block has a smaller acceleration than that, it has a negative acceleration relative to the bottom block.

$$\vec{\mathbf{a}}_{\substack{\text{top rel}\\ \text{bottom}}} = \vec{\mathbf{a}}_{\substack{\text{top rel}\\ \text{ground}}} + \vec{\mathbf{a}}_{\substack{\text{ground rel}\\ \text{bottom}}} = \vec{\mathbf{a}}_{\substack{\text{top rel}\\ \text{ground}}} - \vec{\mathbf{a}}_{\substack{\text{bottom rel}\\ \text{ground}}} = 2.6\,\mathrm{m/s^2}\,\hat{\mathbf{i}} - 5.2\,\mathrm{m/s^2}\,\mathbf{i} = -2.6\,\mathrm{m/s^2}\,\hat{\mathbf{i}}$$

The top block has an acceleration of $\boxed{2.6\,\mathrm{m/s^2}\text{ to the left}}$ relative to the bottom block.

(*d*) No sliding:

$$F_{\substack{x\\ \text{bottom}\\ \text{net}}} = F_{\text{P}} - F_{\substack{\text{fr}\\ \text{bottom}}} = m_{\text{bottom}} a_{\text{bottom}} \quad \rightarrow$$

$$F_{\text{P}} = F_{\substack{\text{fr}\\ \text{bottom}}} + m_{\text{bottom}} a_{\text{bottom}} = F_{\substack{\text{fr}\\ \text{top}}} + m_{\text{bottom}} a_{\text{bottom}} = m_{\text{top}} a_{\text{top}} + m_{\text{bottom}} a_{\text{bottom}} = \left(m_{\text{top}} + m_{\text{bottom}}\right) a$$

$$= (16.0\,\text{kg})\left(5.2\,\text{m/s}^2\right) = \boxed{83\,\text{N}}$$

This is the same as simply assuming that the external force is accelerating the total mass. The internal friction need not be considered if the blocks are not moving relative to each other.

Sliding:

$$F_{\substack{x\\ \text{bottom}\\ \text{net}}} = F_{\text{P}} - F_{\substack{\text{fr}\\ \text{bottom}}} = m_{\text{bottom}} a_{\text{bottom}} \quad \rightarrow$$

$$F_{\text{P}} = F_{\substack{\text{fr}\\ \text{bottom}}} + m_{\text{bottom}} a_{\text{bottom}} = F_{\substack{\text{fr}\\ \text{top}}} + m_{\text{bottom}} a_{\text{bottom}} = m_{\text{top}} a_{\text{top}} + m_{\text{bottom}} a_{\text{bottom}}$$

$$= (4.0\,\text{kg})\left(2.6\,\text{m/s}^2\right) + (12.0\,\text{kg})\left(5.2\,\text{m/s}^2\right) = \boxed{73\,\text{N}}$$

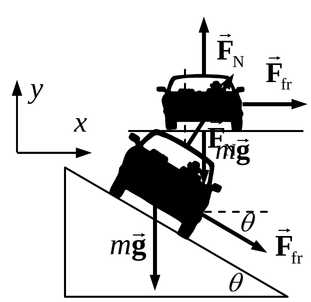
Again this can be interpreted as the external force providing the acceleration for each block. The internal friction need not be considered.

38. The centripetal acceleration of a rotating object is given by $a_{\text{R}} = v^2/r$.

$$v = \sqrt{a_{\text{R}} r} = \sqrt{\left(1.25\times10^5 g\right) r} = \sqrt{\left(1.25\times10^5\right)\left(9.80\,\text{m/s}^2\right)\left(8.00\times10^{-2}\,\text{m}\right)} = 3.13\times10^2\ \text{m/s}.$$

$$\left(3.13\times10^2\ \text{m/s}\right)\left(\frac{1\ \text{rev}}{2\pi\left(8.00\times10^{-2}\text{m}\right)}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \boxed{3.74\times10^4\,\text{rpm}}$$

59. Since the curve is designed for a speed of 85 km/h, traveling at that speed would mean no friction is needed to round the curve. From Example 5-15 in the textbook, the no-friction banking angle is given by



$$\theta = \tan^{-1}\frac{v^2}{rg} = \tan^{-1}\frac{\left[(85\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{(68\,\text{m})\left(9.80\,\text{m/s}^2\right)} = 39.91^\circ$$

Driving at a higher speed with the same radius means that more centripetal force will be required than is present by the normal force alone. That extra centripetal force will be supplied by a force of static friction, downward along the incline, as shown in the first free-body diagram for the car on the incline. Write Newton's second law in both the *x* and *y* directions. The car will have no acceleration in the *y* direction, and centripetal acceleration in the *x* direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{\text{fr}} = \mu_s F_{\text{N}}$.

$$\sum F_y = F_{\text{N}}\cos\theta - mg - F_{\text{fr}}\sin\theta = 0 \quad \rightarrow \quad F_{\text{N}}\cos\theta - \mu_s F_{\text{N}}\sin\theta = mg \quad \rightarrow$$

$$F_{\text{N}} = \frac{mg}{\left(\cos\theta - \mu_s\sin\theta\right)}$$

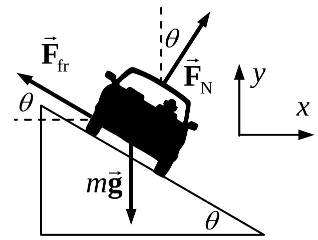
$$\sum F_x = F_{\text{N}}\sin\theta + F_{\text{fr}}\cos\theta = mv^2/r \quad \rightarrow \quad F_{\text{N}}\sin\theta + \mu_s F_{\text{N}}\cos\theta = mv^2/r \quad \rightarrow$$

$$F_{\text{N}} = \frac{mv^2/r}{\left(\sin\theta + \mu_s\cos\theta\right)}$$

Equate the two expressions for the normal force, and solve for the speed.

$$\frac{mv^2/r}{(\sin\theta+\mu_s\cos\theta)}=\frac{mg}{(\cos\theta-\mu_s\sin\theta)} \quad \rightarrow$$

$$v=\sqrt{rg\frac{(\sin\theta+\mu_s\cos\theta)}{(\cos\theta-\mu_s\sin\theta)}}=\sqrt{(68\,\text{m})(9.80\,\text{m/s}^2)\frac{(\sin 39.91^\circ+0.30\cos 39.91^\circ)}{(\cos 39.91^\circ-0.30\sin 39.91^\circ)}}=32\,\text{m/s}$$



Now for the slowest possible speed. Driving at a slower speed with the same radius means that less centripetal force will be required than that supplied by the normal force. That decline in centripetal force will be supplied by a force of static friction, upward along the incline, as shown in the second free-body diagram for the car on the incline. Write Newton's second law in both the *x* and *y* directions. The car will have no acceleration in the *y* direction, and centripetal acceleration in the *x* direction. We also assume that the car is on the verge of skidding, so that the static frictional force has its maximum value of $F_{\text{fr}}=\mu_s F_{\text{N}}$.

$$\sum F_y = F_{\text{N}}\cos\theta - mg + F_{\text{fr}}\sin\theta = 0 \quad \rightarrow$$

$$F_{\text{N}}\cos\theta+\mu_s F_{\text{N}}\sin\theta = mg \quad \rightarrow \quad F_{\text{N}}=\frac{mg}{(\cos\theta+\mu_s\sin\theta)}$$

$$\sum F_x = F_{\text{N}}\sin\theta - F_{\text{fr}}\cos\theta = mv^2/r \quad \rightarrow \quad F_{\text{N}}\sin\theta-\mu_s F_{\text{N}}\cos\theta = mv^2/r \quad \rightarrow$$

$$F_{\text{N}}=\frac{mv^2/r}{(\sin\theta-\mu_s\cos\theta)}$$

Equate the two expressions for the normal force, and solve for the speed.

$$\frac{mv^2/r}{(\sin\theta-\mu_s\cos\theta)}=\frac{mg}{(\cos\theta+\mu_s\sin\theta)} \quad \rightarrow$$

$$v=\sqrt{rg\frac{(\sin\theta-\mu_s\cos\theta)}{(\cos\theta+\mu_s\sin\theta)}}=\sqrt{(68\,\text{m})(9.80\,\text{m/s}^2)\frac{(\sin 39.91^\circ-0.30\cos 39.91^\circ)}{(\cos 39.91^\circ+0.30\sin 39.91^\circ)}}=17\,\text{m/s}$$

Thus the range is $\boxed{17\,\text{m/s}\le v\le 32\,\text{m/s}}$, which is $\boxed{61\,\text{km/h}\le v\le 115\,\text{km/h}}$.

66. (*a*) The terminal velocity is given by Eq. 5-9. This can be used to find the value of *b*.

$$v_{\text{T}}=\frac{mg}{b} \quad \rightarrow \quad b=\frac{mg}{v_{\text{T}}}=\frac{(3\times10^{-5}\,\text{kg})(9.80\,\text{m/s}^2)}{(9\,\text{m/s})}=3.27\times10^{-5}\,\text{kg/s}\approx\boxed{3\times10^{-5}\,\text{kg/s}}$$

(*b*) From Example 5-17, the time required for the velocity to reach 63% of terminal velocity is the time constant, $\tau=m/b$.

$$\tau=\frac{m}{b}=\frac{3\times10^{-5}\,\text{kg}}{3.27\times10^{-5}\,\text{kg/s}}=0.917\,\text{s}\approx\boxed{1\,\text{s}}$$

68. The net force on the falling object, taking downward as positive, will be $\sum F = mg - bv^2 = ma$.

(*a*) The terminal velocity occurs when the acceleration is 0.

$$mg-bv^2=ma \quad \rightarrow \quad mg-bv_{\text{T}}^2=0 \quad \rightarrow \quad \boxed{v_{\text{T}}=\sqrt{mg/b}}$$

(*b*) $v_{\mathrm{T}} = \sqrt{\dfrac{mg}{b}} \quad \rightarrow \quad b = \dfrac{mg}{v_{\mathrm{T}}^2} = \dfrac{(75\,\mathrm{kg})(9.80\,\mathrm{m/s^2})}{(60\,\mathrm{m/s})^2} = \boxed{0.2\,\mathrm{kg/m}}$

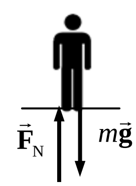
(*c*) The curve would be qualitatively like Fig. 5-27, because the speed would increase from 0 to the terminal velocity, asymptotically. But this curve would be ABOVE the one in Fig. 5-27, because the friction force increases more rapidly. For Fig. 5-27, if the speed doubles, the friction force doubles. But in this case, if the speed doubles, the friction force would increase by a factor of 4, bringing the friction force closer to the weight of the object in a shorter period of time.

80. Since mass *m* is dangling, the tension in the cord must be equal to the weight of mass *m*, and so $F_{\mathrm{T}} = mg$. That same tension is in the other end of the cord, maintaining the circular motion of mass *M*, and so $F_{\mathrm{T}} = F_{\mathrm{R}} = Ma_{\mathrm{R}} = M\,v^2/r$. Equate the expressions for tension and solve for the velocity.

$$M\,v^2/r = mg \quad \rightarrow \quad v = \boxed{\sqrt{mgR/M}}$$

CHAPTER 6

$\boxed{25.}$ Consider a free-body diagram of yourself in the elevator. $\vec{\mathbf{F}}_{\mathrm{N}}$ is the force of the scale pushing up on you, and reads the normal force. Since the scale reads 76 kg, if it were calibrated in Newtons, the normal force would be $F_{\mathrm{N}} = (76\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 744.8\,\mathrm{N}$. Write Newton's second law in the vertical direction, with upward as positive.



$$\sum F = F_{\mathrm{N}} - mg = ma \quad \rightarrow \quad a = \frac{F_{\mathrm{N}} - mg}{m} = \frac{744.8\,\mathrm{N} - (65\,\mathrm{kg})(9.80\,\mathrm{m/s^2})}{65\,\mathrm{kg}} = \boxed{1.7\,\mathrm{m/s^2}\ \text{upward}}$$

Since the acceleration is positive, the acceleration is upward.

27. The speed of an object in a circular orbit of radius *r* around mass *M* is given in Example 6-6 by $v = \sqrt{GM/r}$, and is also given by $v = 2\pi r/T$, where *T* is the period of the orbiting object. Equate the two expressions for the speed and solve for *T*.

$$\sqrt{G\frac{M}{r}} = \frac{2\pi r}{T} \quad \rightarrow$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(1.86\times10^6\,\mathrm{m})^3}{(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2})(7.35\times10^{22}\,\mathrm{m})}} = \boxed{7.20\times10^3\,\mathrm{s} \approx 120\,\mathrm{min}}$$