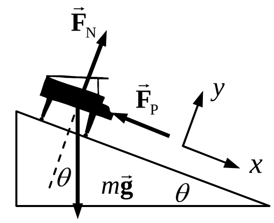
CHAPTER 7: Work and Energy

11. The piano is moving with a constant velocity down the plane. $\vec{\mathbf{F}}_P$ is the force of the man pushing on the piano.



(*a*) Write Newton's second law on each direction for the piano, with an acceleration of 0.

$$\sum F_y = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_P = 0 \quad \rightarrow$$

$$F_P = mg\sin\theta = mg\sin\theta$$

$$= (380\,\text{kg})(9.80\,\text{m/s}^2)(\sin 27°) = 1691\,\text{N} \approx \boxed{1700\,\text{N}}$$

(*b*) The work done by the man is the work done by $\vec{\mathbf{F}}_P$. The angle between $\vec{\mathbf{F}}_P$ and the direction of motion is $180°$. Use Eq. 7-1.

$$W_P = F_P d\cos 180° = -(1691\,\text{N})(3.9\,\text{m}) = -6595\,\text{J} \approx \boxed{-6600\,\text{J}}.$$

(*c*) The angle between the force of gravity and the direction of motion is $63°$. Calculate the work done by gravity.

$$W_G = F_G d\cos 63° = mgd\cos 63° = (380\,\text{kg})(9.80\,\text{m/s}^2)(3.9\,\text{m})\cos 63°$$

$$= 6594\,\text{N} \approx \boxed{6600\,\text{J}}$$

(*d*) Since the piano is not accelerating, the net force on the piano is 0, and so the net work done on the piano is also 0. This can also be seen by adding the two work amounts calculated.

$$W_{net} = W_P + W_G = -6.6\times10^3\,\text{J} + 6.6\times10^3\,\text{J} = \boxed{0\,\text{J}}$$

40. The work done will be the area under the F_x vs. x graph.

(*a*) From $x = 0.0$ to $x = 10.0\,\text{m}$, the shape under the graph is trapezoidal. The area is

$$W_a = (400\,\text{N})\tfrac{1}{2}(10\,\text{m} + 4\,\text{m}) = \boxed{2800\,\text{J}}.$$

(*b*) From $x = 10.0\,\text{m}$ to $x = 15.0\,\text{m}$, the force is in the opposite direction from the direction of motion, and so the work will be negative. Again, since the shape is trapezoidal, we find

$$W_a = (-200\,\text{N})\tfrac{1}{2}(5\,\text{m} + 2\,\text{m}) = -700\,\text{J}.$$

Thus the total work from $x = 0.0$ to $x = 15.0\,\text{m}$ is $2800\text{ J} - 700\text{ J} = \boxed{2100\,\text{J}}$.

52. The work done on the electron is equal to the change in its kinetic energy.

$$W = \Delta K = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = 0 - \tfrac{1}{2}(9.11\times10^{-31}\,\text{kg})(1.40\times10^6\,\text{m/s})^2 = \boxed{-8.93\times10^{-19}\,\text{J}}$$

Note that the work is negative since the electron is slowing down.

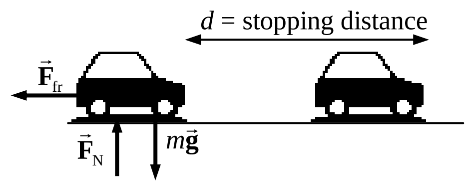
$\boxed{55.}$ The force of the ball on the glove will be the opposite of the force of the glove on the ball, by Newton's third law. Both objects have the same displacement, and so the work done on the glove is opposite the work done on the ball. The work done on the ball is equal to the change in the kinetic energy of the ball.

$$W_{\text{on ball}} = (K_2 - K_1)_{\text{ball}} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = 0 - \tfrac{1}{2}(0.145\,\text{kg})(32\,\text{m/s})^2 = -74.24\,\text{J}$$

So $W_{\text{on glove}} = 74.24\,\text{J}$. But $W_{\text{on glove}} = F_{\text{on glove}} d\cos 0°$, because the force on the glove is in the same direction as the motion of the glove.

$$74.24\,\text{J} = F_{\text{on glove}}\left(0.25\,\text{m}\right) \quad\rightarrow\quad F_{\text{on glove}} = \frac{74.24\,\text{J}}{0.25\,\text{m}} = \boxed{3.0\times10^{2}\,\text{N}}$$, in the direction of the original velocity of the ball.

65. The work needed to stop the car is equal to the change in the car's kinetic energy. That work comes from the force of friction on the car, which is assumed to be static friction since the driver locked the brakes. Thus $F_{\text{fr}} = \mu_k F_{\text{N}}$. Since the car is on a level surface, the normal force is equal to the car's weight, and so $F_{\text{fr}} = \mu_k mg$ if it is on a level surface. See the diagram for the car. The car is traveling to the right.



$$W = \Delta K \quad\rightarrow\quad F_{\text{fr}} d\cos 180^{\circ} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \quad\rightarrow\quad -\mu_k mgd = 0 - \tfrac{1}{2}mv_1^2 \quad\rightarrow$$

$$v_1 = \sqrt{2\mu_k gd} = \sqrt{2\left(0.38\right)\left(9.80\,\text{m/s}^2\right)\left(98\,\text{m}\right)} = \boxed{27\,\text{m/s}}$$

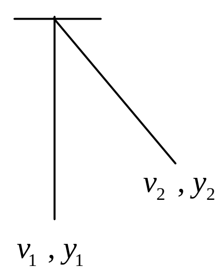
The mass does not affect the problem, since both the change in kinetic energy and the work done by friction are proportional to the mass. The mass cancels out of the equation.

CHAPTER 8: Conservation of Energy

6. Assume that all of the kinetic energy of the car becomes potential energy of the compressed spring.

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}kx_{\text{final}}^2 \quad\rightarrow\quad k = \frac{mv_0^2}{x_{\text{final}}^2} = \frac{\left(1200\,\text{kg}\right)\left[\left(75\,\text{km/h}\right)\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{\left(2.2\,\text{m}\right)^2} = \boxed{1.1\times10^{5}\,\text{N/m}}$$

12. The only forces acting on Jane are gravity and the vine tension. The tension pulls in a centripetal direction, and so can do no work – the tension force is perpendicular at all times to her motion. So Jane's mechanical energy is conserved. Subscript 1 represents Jane at the point where she grabs the vine, and subscript 2 represents Jane at the highest point of her swing. The ground is the zero location for gravitational potential energy $\left(y=0\right)$. We have $v_1 = 5.0\,\text{m/s}$, $y_1 = 0$, and $v_2 = 0$ (top of swing). Solve for y_2, the height of her swing.



$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 + 0 = 0 + mgy_2 \quad\rightarrow$$

$$y_2 = \frac{v_1^2}{2g} = \frac{\left(5.0\,\text{m/s}\right)^2}{2\left(9.80\,\text{m/s}^2\right)} = 1.276\,\text{m} \approx \boxed{1.3\,\text{m}}$$

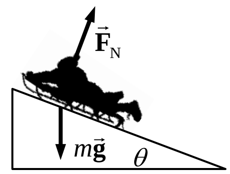
$\boxed{\text{No}}$, the length of the vine does not enter into the calculation, unless the vine is less than 0.65 m long. If that were the case, she could not rise 1.3 m high.

$\boxed{13.}$ We assume that all the forces on the jumper are conservative, so that the mechanical energy of the jumper is conserved. Subscript 1 represents the jumper at the bottom of the jump, and subscript 2 represents the jumper at the top of the jump. Call the ground the zero location for gravitational potential energy $(y=0)$. We have $y_1=0,\ v_2=0.70\,\text{m/s}$, and $y_2=2.10\,\text{m}$. Solve for v_1, the speed at the bottom.

$$\tfrac{1}{2}mv_1^2+mgy_1=\tfrac{1}{2}mv_2^2+mgy_2 \ \rightarrow\ \tfrac{1}{2}mv_1^2+0=\tfrac{1}{2}mv_2^2+mgy_2 \ \rightarrow$$

$$v_1=\sqrt{v_2^2+2gy_2}=\sqrt{(0.70\,\text{m/s})^2+2(9.80\,\text{m/s}^2)(2.10\ \text{m})}=6.454\,\text{m/s}\approx\boxed{6.5\,\text{m/s}}$$

14. The forces on the sled are gravity and the normal force. The normal force is perpendicular to the direction of motion, and so does no work. Thus the sled's mechanical energy is conserved. Subscript 1 represents the sled at the bottom of the hill, and subscript 2 represents the sled at the top of the hill. The ground is the zero location for gravitational potential energy $(y=0)$. We have $y_1=0$, $v_2=0$, and $y_2=1.12\,\text{m}$. Solve for v_1, the speed at the bottom.



$$\tfrac{1}{2}mv_1^2+mgy_1=\tfrac{1}{2}mv_2^2+mgy_2 \ \rightarrow\ \tfrac{1}{2}mv_1^2+0=0+mgy_2 \ \rightarrow$$

$$v_1=\sqrt{2gy_2}=\sqrt{2(9.80\,\text{m/s}^2)(1.12\,\text{m})}=\boxed{4.69\,\text{m/s}}$$

Notice that the angle is not used in the calculation.

40. Since there is friction in this problem, there will be energy dissipated by friction.

$$E_\text{friction}+\Delta K+\Delta U=0 \ \rightarrow\ E_\text{friction}=-\Delta K-\Delta U=\tfrac{1}{2}m\left(v_1^2-v_2^2\right)+mg\left(y_1-y_2\right)$$

$$=\tfrac{1}{2}(56\,\text{kg})\left[0-(11.0\,\text{m/s})^2\right]+(56\,\text{kg})(9.80\,\text{m/s}^2)(230\,\text{m})=\boxed{1.2\times10^5\,\text{J}}$$

47. The escape velocity is given by Eq. 8-19.

$$v_{\underset{\text{A}}{\text{esc}}}=\sqrt{\frac{2M_\text{A}G}{r_\text{A}}} \quad v_{\underset{\text{B}}{\text{esc}}}=\sqrt{\frac{2M_\text{B}G}{r_\text{B}}} \quad v_{\underset{\text{A}}{\text{esc}}}=2v_{\underset{\text{B}}{\text{esc}}} \ \rightarrow\ \sqrt{\frac{2M_\text{A}G}{r_\text{A}}}=2\sqrt{\frac{2M_\text{B}G}{r_\text{B}}} \ \rightarrow$$

$$\frac{2M_\text{A}G}{r_\text{A}}=2\left(\frac{2M_\text{B}G}{r_\text{B}}\right) \ \rightarrow\ \boxed{\frac{r_\text{A}}{r_\text{B}}=\frac{1}{4}}$$

62. The work necessary to lift the piano is the work done by an upward force, equal in magnitude to the weight of the piano. Thus $W=Fd\cos 0°=mgh$. The average power output required to lift the piano is the work done divided by the time to lift the piano.

$$P=\frac{W}{t}=\frac{mgh}{t} \ \rightarrow\ t=\frac{mgh}{P}=\frac{(335\ \text{kg})(9.80\,\text{m/s}^2)(16.0\ \text{m})}{1750\ \text{W}}=\boxed{30.0\,\text{s}}$$

$\boxed{85.}$ (*a*) The tension in the cord is perpendicular to the path at all times, and so the tension in the cord does not do any work on the ball. Thus only gravity does work on the ball, and so the mechanical energy of the ball is conserved. Subscript 1 represents the ball when it is horizontal, and subscript 2 represents the ball at the lowest point on its path. The lowest point on the path is the zero location for potential energy $(y=0)$. We have $v_1=0$, $y_1=l$, and $y_2=0$. Solve for v_2.

$$E_1 = E_2 \quad \to \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \to \quad mgl = \tfrac{1}{2}mv_2^2 \quad \to \quad v_2 = \boxed{\sqrt{2gl}}$$

(*b*) Use conservation of energy, to relate points 2 and 3. Point 2 is as described above. Subscript 3 represents the ball at the top of its circular path around the peg. The lowest point on the path is the zero location for potential energy $(y = 0)$. We have $v_2 = \sqrt{2gl}$, $y_2 = 0$, and $y_3 = 2(l - h) = 2(l - 0.80l) = 0.40l$. Solve for v_3.

$$E_2 = E_3 \quad \to \quad \tfrac{1}{2}mv_2^2 + mgy_2 = \tfrac{1}{2}mv_3^2 + mgy_3 \quad \to \quad \tfrac{1}{2}m(2gl) = \tfrac{1}{2}mv_3^2 + mg(0.40l) \quad \to$$

$$\boxed{v_3 = \sqrt{1.2gl}}$$