CHAPTER 9: Linear Momentum

9. Consider the motion in one dimension, with the positive direction being the direction of motion of the first car. Let A represent the first car and B represent the second car. Momentum will be conserved in the collision. Note that $v_B = 0$.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad m_A v_A + m_B v_B = (m_A + m_B) v' \quad \rightarrow$$

$$m_B = \frac{m_A (v_A - v')}{v'} = \frac{(7700\,\text{kg})(18\,\text{m/s} - 5.0\,\text{m/s})}{5.0\,\text{m/s}} = \boxed{2.0 \times 10^4\,\text{kg}}$$

13. The throwing of the package is a momentum-conserving action, if the water resistance is ignored. Let A represent the boat and child together, and let B represent the package. Choose the direction that the package is thrown as the positive direction. Apply conservation of momentum, with the initial velocity of both objects being 0.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad (m_A + m_B) v = m_A v'_A + m_B v'_B \quad \rightarrow$$

$$v'_A = -\frac{m_B v'_B}{m_A} = -\frac{(5.70\,\text{kg})(10.0\,\text{m/s})}{(24.0\,\text{kg} + 35.0\,\text{kg})} = \boxed{-0.966\,\text{m/s}}$$

The boat and child move in the opposite direction as the thrown package, as indicated by the negative velocity.

15. Momentum will be conserved in one dimension in the explosion. Let A represent the fragment with the larger kinetic energy.

$$p_{\text{initial}} = p_{\text{final}} \quad \rightarrow \quad 0 = m_A v'_A + m_B v'_B \quad \rightarrow \quad v'_B = -\frac{m_A v'_A}{m_B}$$

$$K_A = 2K_B \quad \rightarrow \quad \tfrac{1}{2} m_A v'^2_A = 2\left(\tfrac{1}{2} m_B v'^2_B\right) = m_B \left(-\frac{m_A v'_A}{m_B}\right)^2 \quad \rightarrow \quad \frac{m_A}{m_B} = \boxed{\frac{1}{2}}$$

The fragment with the larger kinetic energy has half the mass of the other fragment.

25. The impulse given the ball is the change in the ball's momentum. From the symmetry of the problem, the vertical momentum of the ball does not change, and so there is no vertical impulse. Call the direction AWAY from the wall the positive direction for momentum perpendicular to the wall.

$$\Delta p_\perp = m v_{\perp_{\text{final}}} - m v_{\perp_{\text{initial}}} = m\left(v \sin 45^\circ - -v \sin 45^\circ\right) = 2 m v \sin 45^\circ$$

$$= 2\left(6.0 \times 10^{-2}\,\text{km}\right)(25\,\text{m/s}) \sin 45^\circ = \boxed{2.1\,\text{kg}\cdot\text{m/s, to the left}}$$

28. (*a*) The impulse given the ball is the area under the *F* vs. *t* graph. Approximate the area as a triangle of "height" 250 N, and "width" 0.04 sec. A width slightly smaller than the base was chosen to compensate for the "inward" concavity of the force graph.

$$\Delta p = \tfrac{1}{2}(250\ \text{N})(0.04\ \text{s}) = \boxed{5\,\text{N}\cdot\text{s}}$$

(*b*) The velocity can be found from the change in momentum. Call the positive direction the direction of the ball's travel after being served.

$$\Delta p = m \Delta v = m\left(v_f - v_i\right) \quad \rightarrow \quad v_f = v_i + \frac{\Delta p}{m} = 0 + \frac{5\,\text{N}\cdot\text{s}}{6.0 \times 10^{-2}\,\text{kg}} = \boxed{80\,\text{m/s}}$$

39. The one-dimensional stationary target elastic collision is analyzed in Example 9-8. The fraction of kinetic energy lost is found as follows.

$$\frac{K_{\text{A}_{\text{inital}}} - K_{\text{A}_{\text{final}}}}{K_{\text{A}_{\text{inital}}}} = \frac{K_{\text{B}_{\text{final}}}}{K_{\text{A}_{\text{inital}}}} = \frac{\frac{1}{2}m_{\text{B}}v_{\text{B}}'^2}{\frac{1}{2}m_{\text{A}}v_{\text{A}}^2} = \frac{m_{\text{B}}\left[v_{\text{A}}\left(\frac{2m_{\text{A}}}{m_{\text{A}}+m_{\text{B}}}\right)\right]^2}{m_{\text{A}}v_{\text{A}}^2} = \frac{4m_{\text{A}}m_{\text{B}}}{\left(m_{\text{A}}+m_{\text{B}}\right)^2}$$

(*a*) $\frac{4m_{\text{A}}m_{\text{B}}}{\left(m_{\text{A}}+m_{\text{B}}\right)^2} = \frac{4(1.01)(1.01)}{(1.01+1.01)^2} = \boxed{1.00}$

All the initial kinetic energy is lost by the neutron, as expected for the target mass equal to the incoming mass.

(*b*) $\frac{4m_{\text{A}}m_{\text{B}}}{\left(m_{\text{A}}+m_{\text{B}}\right)^2} = \frac{4(1.01)(2.01)}{(1.01+2.01)^2} = \boxed{0.890}$

(*c*) $\frac{4m_{\text{A}}m_{\text{B}}}{\left(m_{\text{A}}+m_{\text{B}}\right)^2} = \frac{4(1.01)(12.00)}{(1.01+12.00)^2} = \boxed{0.286}$

(*d*) $\frac{4m_{\text{A}}m_{\text{B}}}{\left(m_{\text{A}}+m_{\text{B}}\right)^2} = \frac{4(1.01)(208)}{(1.01+208)^2} = \boxed{0.0192}$

Since the target is quite heavy, almost no kinetic energy is lost. The incoming particle "bounces off" the heavy target, much as a rubber ball bounces off a wall with approximately no loss in speed.

50. The swinging motion will conserve mechanical energy. Take the zero level for gravitational potential energy to be at the bottom of the arc. For the pendulum to swing exactly to the top of the arc, the potential energy at the top of the arc must be equal to the kinetic energy at the bottom.

$$K_{\text{bottom}} = U_{\text{top}} \rightarrow \tfrac{1}{2}(m+M)V_{\text{bottom}}^2 = (m+M)g(2L) \rightarrow V_{\text{bottom}} = 2\sqrt{gL}$$

Momentum will be conserved in the totally inelastic collision at the bottom of the arc. We assume that the pendulum does not move during the collision process.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow mv = (m+M)V_{\text{bottom}} \rightarrow v = \frac{m+M}{m} = \boxed{2\frac{m+M}{m}\sqrt{gL}}$$

56. Write momentum conservation in the x and y directions, and kinetic energy conservation. Note that both masses are the same. We allow $\vec{\mathbf{v}}_{\text{A}}'$ to have both x and y components.

$$p_x: \quad mv_{\text{B}} = mv_{\text{A}x}' \rightarrow v_{\text{B}} = v_{\text{A}x}'$$

$$p_y: \quad mv_{\text{A}} = mv_{\text{A}y}' + mv_{\text{B}}' \rightarrow v_{\text{A}} = v_{\text{A}y}' + v_{\text{B}}'$$

$$K: \quad \tfrac{1}{2}mv_{\text{A}}^2 + \tfrac{1}{2}mv_{\text{B}}^2 = \tfrac{1}{2}mv_{\text{A}}'^2 + \tfrac{1}{2}mv_{\text{B}}'^2 \rightarrow v_{\text{A}}^2 + v_{\text{B}}^2 = v_{\text{A}}'^2 + v_{\text{B}}'^2$$

Substitute the results from the momentum equations into the kinetic energy equation.

$$\left(v_{\text{A}y}' + v_{\text{B}}'\right)^2 + \left(v_{\text{A}x}'\right)^2 = v_{\text{A}}'^2 + v_{\text{B}}'^2 \rightarrow v_{\text{A}y}'^2 + 2v_{\text{A}y}'^2v_{\text{B}}' + v_{\text{B}}'^2 + v_{\text{A}y}'^2 = v_{\text{A}}'^2 + v_{\text{B}}'^2 \rightarrow$$

$$v_{\text{A}}'^2 + 2v_{\text{A}y}'^2v_{\text{B}}' + v_{\text{B}}'^2 = v_{\text{A}}'^2 + v_{\text{B}}'^2 \rightarrow 2v_{\text{A}y}'^2v_{\text{B}}' = 0 \rightarrow v_{\text{A}y}' = 0 \text{ or } v_{\text{B}}' = 0$$

Since we are given that $v_{\text{B}}' \neq 0$, we must have $v_{\text{A}y}' = 0$. This means that the final direction of A is the $\boxed{x \text{ direction}}$. Put this result into the momentum equations to find the final speeds.

$$v_{\text{A}}' = v_{\text{A}x}' = v_{\text{B}} = \boxed{3.7\,\text{m/s}} \qquad v_{\text{B}}' = v_{\text{A}} = \boxed{2.0\,\text{m/s}}$$

64. By the symmetry of the problem, since the centers of the cubes are along a straight line, the vertical CM coordinate will be 0, and the depth CM coordinate will be 0. The only CM coordinate to calculate is the one along the straight line joining the centers. The mass of each cube will be the

volume times the density, and so $m_1 = \rho\left(l_0\right)^3$, $m_2 = \rho\left(2l_0\right)^3$, $m_3 = \rho\left(3l_0\right)^3$. Measuring from the left edge of the smallest block, the locations of the CMs of the individual cubes are $x_1 = \frac{1}{2}l_0$, $x_2 = 2l_0$, $x_3 = 4.5l_0$. Use Eq. 9-10 to calculate the CM of the system.

$$x_{\text{CM}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{\rho l_0^3\left(\frac{1}{2}l_0\right) + 8\rho l_0^3\left(2l_0\right) + 27\rho l_0^3\left(4.5l_0\right)}{\rho l_0^3 + 8\rho l_0^3 + 27\rho l_0^3}$$

$$= \boxed{3.8\,l_0 \text{ from the left edge of the smallest cube}}$$

89. This is a ballistic "pendulum" of sorts, similar to Example 9-11 in the textbook. There is no difference in the fact that the block and bullet are moving vertically instead of horizontally. The collision is still totally inelastic and conserves momentum, and the energy is still conserved in the rising of the block and embedded bullet after the collision. So we simply quote the equation from that example.

$$v = \frac{m+M}{m}\sqrt{2gh} \quad \rightarrow$$

$$h = \frac{1}{2g}\left(\frac{mv}{m+M}\right)^2 = \frac{1}{2\left(9.80\,\text{m/s}^2\right)}\left(\frac{\left(0.0240\,\text{kg}\right)\left(310\,\text{m/s}\right)}{0.0240\,\text{kg} + 1.40\text{ kg}}\right)^2 = \boxed{1.4\,\text{m}}$$

100. (*a*) Use conservation of energy to find the speed of mass *m* before the collision. The potential energy at the starting point is all transformed into kinetic energy just before the collision.

$$mgh_{\text{A}} = \tfrac{1}{2}mv_{\text{A}}^2 \quad \rightarrow \quad v_{\text{A}} = \sqrt{2gh_{\text{A}}} = \sqrt{2\left(9.80\,\text{m/s}^2\right)\left(3.60\text{ m}\right)} = 8.40\,\text{m/s}$$

Use Eq. 9-8 to obtain a relationship between the velocities, noting that $v_B = 0$.

$$v_{\text{A}} - v_{\text{B}} = v'_{\text{B}} - v'_{\text{A}} \quad \rightarrow \quad v'_{\text{B}} = v'_{\text{A}} + v_{\text{A}}$$

Apply momentum conservation for the collision, and substitute the result from Eq. 9-8.

$$mv_{\text{A}} = mv'_{\text{A}} + Mv'_{\text{B}} = mv'_{\text{A}} + M\left(v_{\text{A}} + v'_{\text{A}}\right) \quad \rightarrow$$

$$v'_{\text{A}} = \frac{m-M}{m+M}v_{\text{A}} = \left(\frac{2.20\text{ kg} - 7.00\text{ kg}}{9.20\text{ kg}}\right)\left(8.4\,\text{m/s}\right) = -4.38\,\text{m/s} \approx \boxed{-4.4\,\text{m/s}}$$

$$v'_{\text{B}} = v'_{\text{A}} + v_{\text{A}} = -4.4\,\text{m/s} + 8.4\,\text{m/s} = \boxed{4.0\,\text{m/s}}$$

(*b*) Again use energy conservation to find the height to which mass *m* rises after the collision. The kinetic energy of *m* immediately after the collision is all transformed into potential energy. Use the angle of the plane to change the final height into a distance along the incline.

$$\tfrac{1}{2}mv'^2_{\text{A}} = mgh'_{\text{A}} \quad \rightarrow \quad h'_{\text{A}} = \frac{v'^2_{\text{A}}}{2g}$$

$$d'_{\text{A}} = \frac{h'_{\text{A}}}{\sin 30^\circ} = \frac{v'^2_{\text{A}}}{2g\sin 30^\circ} = \frac{\left(-4.38\,\text{m/s}\right)^2}{2\left(9.8\,\text{m/s}^2\right)g\sin 30^\circ} = 1.96\text{ m} \approx \boxed{2.0\text{ m}}$$