Phys101 Mastering Physics Assignment #7

CHAPTER 10: Rotational Motion

1. (*a*) $(45.0°)(2\pi \text{ rad}/360°) = \boxed{\pi/4\,\text{rad}} = \boxed{0.785\,\text{rad}}$

   (*b*) $(60.0°)(2\pi \text{ rad}/360°) = \boxed{\pi/3\,\text{rad}} = \boxed{1.05\,\text{rad}}$

   (*c*) $(90.0°)(2\pi \text{ rad}/360°) = \boxed{\pi/2\,\text{rad}} = \boxed{1.57\,\text{rad}}$

   (*d*) $(360.0°)(2\pi \text{ rad}/360°) = \boxed{2\pi\,\text{rad}} = \boxed{6.283\,\text{rad}}$

   (*e*) $(445°)(2\pi \text{ rad}/360°) = \boxed{89\pi/36\,\text{rad}} = \boxed{7.77\,\text{rad}}$

4. The initial angular velocity is $\omega_o = \left(6500\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = 681\,\text{rad/s}$. Use the definition of angular acceleration.

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0 - 681\,\text{rad/s}}{4.0\text{ s}} = \boxed{-170\,\text{rad/s}^2}$$

11. The centripetal acceleration is given by $a = \omega^2 r$. Solve for the angular velocity.

$$\omega = \sqrt{\frac{a}{r}} = \sqrt{\frac{(100,000)(9.80\,\text{m/s}^2)}{0.070\text{ m}}} = 3741\frac{\text{rad}}{\text{s}}\left(\frac{1\,\text{rev}}{2\pi\,\text{rad}}\right)\left(\frac{60\,\text{s}}{1\text{ min}}\right) = \boxed{3.6\times10^4\,\text{rpm}}$$

12. Convert the rpm values to angular velocities.

$$\omega_0 = \left(130\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = 13.6\,\text{rad/s}$$

$$\omega = \left(280\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = 29.3\,\text{rad/s}$$

(*a*) The angular acceleration is found from Eq. 10-3a.

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{29.3\,\text{rad/s} - 13.6\,\text{rad/s}}{4.0\text{ s}} = 3.93\,\text{rad/s}^2 \approx \boxed{3.9\,\text{rad/s}^2}$$

(*b*) To find the components of the acceleration, the instantaneous angular velocity is needed.

$$\omega = \omega_0 + \alpha t = 13.6\,\text{rad/s} + (3.93\,\text{rad/s}^2)(2.0\,\text{s}) = 21.5\,\text{rad/s}$$

The instantaneous radial acceleration is given by $a_R = \omega^2 r$.

$$a_R = \omega^2 r = (21.5\,\text{rad/s})^2(0.35\,\text{m}) = \boxed{160\,\text{m/s}^2}$$

The tangential acceleration is given by $a_{\tan} = \alpha r$.

$$a_{\tan} = \alpha r = (3.93\,\text{rad/s}^2)(0.35\,\text{m}) = \boxed{1.4\,\text{m/s}^2}$$

16. (*a*) For constant angular acceleration:

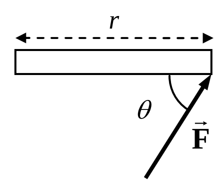
$$\alpha = \frac{\omega - \omega_o}{t} = \frac{1200\,\text{rev/min} - 3500\,\text{rev/min}}{2.5\text{ s}} = \frac{-2300\,\text{rev/min}}{2.5\text{s}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)$$

$$= -96.34\,\text{rad/s}^2 \approx \boxed{-96\,\text{rad/s}^2}$$

(*b*) For the angular displacement, given constant angular acceleration:

$$\theta = \tfrac{1}{2}(\omega_o + \omega)t = \tfrac{1}{2}(3500\,\text{rev/min} + 1200\,\text{rev/min})(2.5\text{ s})\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{98\,\text{rev}}$$

26. The torque is calculated by $\tau = rF\sin\theta$. See the diagram, from the top view.



(*a*) For the first case, $\theta = 90°$.

$$\tau = rF\sin\theta = (0.96\,\text{m})(32\text{ N})\sin 90° = \boxed{31\,\text{m}\cdot\text{N}}$$

(*b*) For the second case, $\theta = 60.0°$.

$$\tau = rF\sin\theta = (0.96\,\text{m})(32\text{ N})\sin 60.0° = \boxed{27\,\text{m}\cdot\text{N}}$$

30. For each torque, use Eq. 10-10c. Take counterclockwise torques to be positive.

(*a*) Each force has a lever arm of 1.0 m.

$$\tau_{\substack{\text{about}\\ \text{C}}} = -(1.0\,\text{m})(56\,\text{N})\sin 30° + (1.0\,\text{m})(52\,\text{N})\sin 60° = \boxed{17\text{m}\cdot\text{N}}$$

(*b*) The force at C has a lever arm of 1.0 m, and the force at the top has a lever arm of 2.0 m.

$$\tau_{\substack{\text{about}\\ \text{P}}} = -(2.0\,\text{m})(56\,\text{N})\sin 30° + (1.0\,\text{m})(65\,\text{N})\sin 45° = \boxed{-10\,\text{m}\cdot\text{N}} \text{ (2 sig fig)}$$

The negative sign indicates a clockwise torque.

38. (*a*) The torque gives angular acceleration to the ball only, since the arm is considered massless. The angular acceleration of the ball is found from the given tangential acceleration.

$$\tau = I\alpha = MR^2\alpha = MR^2\frac{a_{\text{tan}}}{R} = MRa_{\text{tan}} = (3.6\,\text{kg})(0.31\,\text{m})(7.0\,\text{m/s}^2)$$

$$= 7.812\,\text{m}\cdot\text{N} \approx \boxed{7.8\,\text{m}\cdot\text{N}}$$

(*b*) The triceps muscle must produce the torque required, but with a lever arm of only 2.5 cm, perpendicular to the triceps muscle force.

$$\tau = Fr_\perp \quad\rightarrow\quad F = \tau/r_\perp = 7.812\,\text{m}\cdot\text{N}\big/\left(2.5\times10^{-2}\,\text{m}\right) = \boxed{310\,\text{N}}$$

42. The torque supplied is equal to the angular acceleration times the moment of inertia. The angular acceleration is found using Eq. 10-9b, with $\omega_0 = 0$. Use the moment of inertia of a sphere.

$$\theta = \omega_0 + \tfrac{1}{2}\alpha t^2 \quad\rightarrow\quad \alpha = \frac{2\theta}{t^2}\ ;\ \tau = I\alpha = \left(\tfrac{2}{5}Mr_0^2\right)\left(\frac{2\theta}{t^2}\right) \quad\rightarrow$$

$$M = \frac{5\tau t^2}{4r_0^2\theta} = \frac{5(10.8\,\text{m}\cdot\text{N})(15.0\,\text{s})^2}{4(0.36\,\text{m})^2(360\pi\text{ rad})} = \boxed{21\,\text{kg}}$$

48. The torque on the rotor will cause an angular acceleration given by $\alpha = \tau/I$. The torque and angular acceleration will have the opposite sign of the initial angular velocity because the rotor is being brought to rest. The rotational inertia is that of a solid cylinder. Substitute the expressions for angular acceleration and rotational inertia into the equation $\omega^2 = \omega_o^2 + 2\alpha\theta$, and solve for the angular displacement.

$$\omega^2 = \omega_o^2 + 2\alpha\theta \quad \rightarrow \quad \theta = \frac{\omega^2 - \omega_o^2}{2\alpha} = \frac{0 - \omega_o^2}{2(\tau/I)} = \frac{-\omega_o^2}{2\left(\tau/\tfrac{1}{2}MR^2\right)} = \frac{-MR^2\omega_o^2}{4\tau}$$

$$= \frac{-(3.80\,\text{kg})(0.0710\,\text{m})^2\left[\left(10{,}300\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\,\text{rad}}{1\,\text{rev}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)\right]^2}{4(-1.20\,\text{N}\cdot\text{m})} = 4643\,\text{rad}\left(\frac{1\,\text{rev}}{2\pi\,\text{rad}}\right)$$

$$= \boxed{739\,\text{rev}}$$

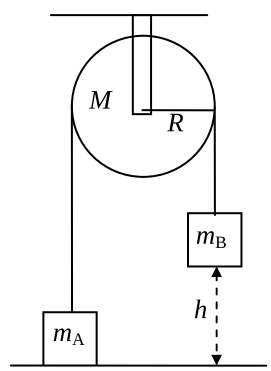
The time can be found from $\theta = \tfrac{1}{2}(\omega_o + \omega)t$.

$$t = \frac{2\theta}{\omega_o + \omega} = \frac{2(739\,\text{rev})}{10{,}300\,\text{rev/min}}\left(\frac{60\,\text{s}}{1\,\text{min}}\right) = \boxed{8.61\,\text{s}}$$

65. The work required is the change in rotational kinetic energy. The initial angular velocity is 0.

$$W = \Delta K_{\text{rot}} = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2 = \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega_f^2 = \tfrac{1}{4}(1640\,\text{kg})(7.50\,\text{m})^2\left(\frac{2\pi\,\text{rad}}{8.00\text{ s}}\right)^2 = \boxed{1.42\times10^4\,\text{J}}$$

67. The only force doing work in this system is gravity, so mechanical energy is conserved. The initial state of the system is the configuration with m_A on the ground and all objects at rest. The final state of the system has m_B just reaching the ground, and all objects in motion. Call the zero level of gravitational potential energy to be the ground level. Both masses will have the same speed since they are connected by the rope. Assuming that the rope does not slip on the pulley, the angular speed of the pulley is related to the speed of the masses by $\omega = v/R$. All objects have an initial speed of 0.



$$E_i = E_f \quad \rightarrow$$

$$\tfrac{1}{2}m_A v_i^2 + \tfrac{1}{2}m_B v_i^2 + \tfrac{1}{2}I\omega_i^2 + m_A g y_{1i} + m_B g y_{2i} = \tfrac{1}{2}m_A v_f^2 + \tfrac{1}{2}m_B v_f^2 + \tfrac{1}{2}I\omega_f^2 + m_A g y_{1f} + m_B g y_{2f}$$

$$m_B g h = \tfrac{1}{2}m_A v_f^2 + \tfrac{1}{2}m_B v_f^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{v_f^2}{R^2}\right) + m_A g h$$

$$v_f = \sqrt{\frac{2(m_B - m_A)gh}{\left(m_A + m_B + \tfrac{1}{2}M\right)}} = \sqrt{\frac{2(38.0\,\text{kg} - 35.0\,\text{kg})(9.80\,\text{m/s}^2)(2.5\,\text{m})}{\left(38.0\text{ kg} + 35.0\text{ kg} + \left(\tfrac{1}{2}\right)3.1\text{ kg}\right)}} = \boxed{1.4\,\text{m/s}}$$

69. Since the lower end of the pole does not slip on the ground, the friction does no work, and so mechanical energy is conserved. The initial energy is the potential energy, treating all the mass as if it were at the CM. The final energy is rotational kinetic energy, for rotation about the point of contact with the ground. The linear velocity of the falling tip of the rod is its angular velocity divided by the length.

$$E_{\text{intial}} = E_{\text{final}} \;\rightarrow\; U_{\text{initial}} = K_{\text{final}} \;\rightarrow\; mgh = \tfrac{1}{2}I\omega^2 \;\rightarrow\; mg\,L/2 = \tfrac{1}{2}\left(\tfrac{1}{3}mL^2\right)\left(v_{\text{end}}/L\right)^2 \;\rightarrow$$

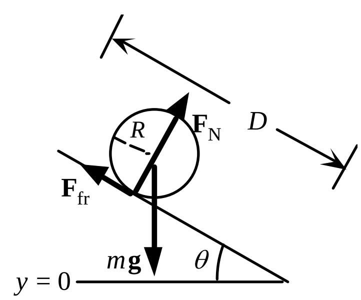
$$v_{\text{end}} = \sqrt{3gL} = \sqrt{3\left(9.80\,\text{m/s}^2\right)\left(2.30\text{ m}\right)} = \boxed{8.22\,\text{m/s}}$$

71. The total kinetic energy is the sum of the translational and rotational kinetic energies. Since the ball is rolling without slipping, the angular velocity is given by $\omega = v/R$. The rotational inertia of a sphere about an axis through its center is $I = \tfrac{2}{5}mR^2$.

$$K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\tfrac{2}{5}mR^2\frac{v^2}{R^2} = \tfrac{7}{10}mv^2$$

$$= 0.7\left(7.3\text{ kg}\right)\left(3.7\,\text{m/s}\right)^2 = \boxed{7.0\times10^1\text{ J}}$$

77. (*a*) Use conservation of mechanical energy. Call the zero level for gravitational potential energy to be the lowest point on which the pipe rolls. Since the pipe rolls without slipping, $\omega = v/R$. See the attached diagram.



$$E_{\text{initial}} = E_{\text{final}} \;\rightarrow\; U_{\text{initial}} = K_{\text{final}} = K_{\text{CM}} + K_{\text{rot}}$$

$$mgD\sin\theta = \tfrac{1}{2}mv^2_{\text{bottom}} + \tfrac{1}{2}I\omega^2_{\text{bottom}}$$

$$= \tfrac{1}{2}mv^2_{\text{bottom}} + \tfrac{1}{2}\left(mR^2\right)\left(\frac{v^2_{\text{bottom}}}{R^2}\right) = mv^2_{\text{bottom}} \;\rightarrow$$

$$v_{\text{bottom}} = \sqrt{gD\sin\theta} = \sqrt{\left(9.80\,\text{m/s}^2\right)\left(5.60\,\text{m}\right)\sin17.5^\circ} = \boxed{4.06\,\text{m/s}}$$

(*b*) The total kinetic energy at the base of the incline is the same as the initial potential energy.

$$K_{\text{final}} = U_{\text{initial}} = mgD\sin\theta = \left(0.545\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(5.60\,\text{m}\right)\sin17.5^\circ = \boxed{8.99\,\text{J}}$$

(*c*) The frictional force supplies the torque for the object to roll without slipping, and the frictional force has a maximum value. Since the object rolls without slipping, $\alpha = a/R$. Use Newton's second law for the directions parallel and perpendicular to the plane, and for the torque, to solve for the coefficient of friction.

$$\sum\tau = F_{\text{fr}}R = I\alpha = mR^2\frac{a}{R} = maR \;\rightarrow\; F_{\text{fr}} = ma$$

$$\sum F_{\perp} = F_{\text{N}} - mg\cos\theta \;\rightarrow\; F_{\text{N}} = mg\cos\theta$$

$$\sum F_{\parallel} = mg\sin\theta - F_{\text{fr}} = ma \;\rightarrow\; F_{\text{fr}} = \tfrac{1}{2}mg\sin\theta$$

$$F_{\text{fr}} \le F_{\substack{\text{static}\\ \text{max}}} \;\rightarrow\; \tfrac{1}{2}mg\sin\theta \le \mu_{\text{s}}F_N = \mu_{\text{s}}mg\cos\theta \;\rightarrow\; \mu_{\text{s}} \ge \tfrac{1}{2}\tan\theta \;\rightarrow$$

$$\mu_{\substack{\text{s}\\ \text{min}}} = \tfrac{1}{2}\tan\theta = \tfrac{1}{2}\tan17.5^\circ = \boxed{0.158}$$