CHAPTER 11: Angular Momentum; General Rotation

8. (*a*) $L = I\omega = \frac{1}{2}MR^2\omega = \frac{1}{2}\left(48\,\text{kg}\right)\left(0.15\,\text{m}\right)^2\left(2.8\frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\,\text{rad}}{1\,\text{rev}}\right) = 9.50\,\text{kg}\cdot\text{m}^2/\text{s} \approx \boxed{9.5\,\text{kg}\cdot\text{m}^2/\text{s}}$

(*b*) If the rotational inertia does not change, then the change in angular momentum is strictly due to a change in angular velocity.

$$\tau = \frac{\Delta L}{\Delta t} = \frac{I\omega_{\text{final}} - I\omega_0}{\Delta t} = \frac{0 - 9.50\,\text{kg}\cdot\text{m}^2/\text{s}}{5.0\text{ s}} = \boxed{-1.9\,\text{m}\cdot\text{N}}$$

The negative sign indicates that the torque is in the opposite direction as the initial angular momentum.

10. The angular momentum of the disk–rod combination will be conserved because there are no external torques on the combination. This situation is a totally inelastic collision, in which the final angular velocity is the same for both the disk and the rod. Subscript 1 represents before the collision, and subscript 2 represents after the collision. The rod has no initial angular momentum.

$$L_1 = L_2 \quad\rightarrow\quad I_1\omega_1 = I_2\omega_2 \quad\rightarrow$$

$$\omega_2 = \omega_1\frac{I_1}{I_2} = \omega_1\frac{I_{\text{disk}}}{I_{\text{disk}} + I_{\text{rod}}} = \omega_1\left[\frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + \frac{1}{12}M\left(2R\right)^2}\right] = \left(3.7\,\text{rev/s}\right)\left(\frac{3}{5}\right) = \boxed{2.2\,\text{rev/s}}$$

11. Since the person is walking radially, no torques will be exerted on the person–platform system, and so angular momentum will be conserved. The person will be treated as a point mass. Since the person is initially at the center, they have no initial rotational inertia.

(*a*) $L_i = L_f \quad\rightarrow\quad I_{\text{platform}}\omega_i = \left(I_{\text{platform}} + I_{\text{person}}\right)\omega_f$

$$\omega_f = \frac{I_{\text{platform}}}{I_{\text{platform}} + mR^2}\omega_i = \frac{920\,\text{kg}\cdot\text{m}^2}{920\,\text{kg}\cdot\text{m}^2 + \left(75\,\text{kg}\right)\left(3.0\,\text{m}\right)^2}\left(0.95\,\text{rad/s}\right) = 0.548\,\text{rad/s} \approx \boxed{0.55\,\text{rad/s}}$$

(*b*) $KE_i = \frac{1}{2}I_{\text{platform}}\omega_i^2 = \frac{1}{2}\left(920\,\text{kg}\cdot\text{m}^2\right)\left(0.95\,\text{rad/s}\right)^2 = \boxed{420\,\text{J}}$

$$KE_f = \frac{1}{2}\left(I_{\text{platform}} + I_{\text{person}}\right)\omega_f^2 = \frac{1}{2}\left(I_{\text{platform}} + m_{\text{person}}r_{\text{person}}^2\right)\omega_f^2$$

$$= \frac{1}{2}\left[920\,\text{kg}\cdot\text{m}^2 + \left(75\,\text{kg}\right)\left(3.0\,\text{m}\right)^2\right]\left(0.548\,\text{rad/s}\right)^2 = 239\,\text{J} \approx \boxed{240\,\text{J}}$$

19. The angular momentum of the person–turntable system will be conserved. Call the direction of the person's motion the positive rotation direction. Relative to the ground, the person's speed will be $v + v_{\text{T}}$, where v is the person's speed relative to the turntable, and v_{T} is the speed of the rim of the turntable with respect to the ground. The turntable's angular speed is $\omega_{\text{T}} = v_{\text{T}}/R$, and the person's angular speed relative to the ground is $\omega_{\text{P}} = \frac{v + v_{\text{T}}}{R} = \frac{v}{R} + \omega_{\text{T}}$. The person is treated as a point particle for calculation of the moment of inertia.

$$L_i = L_f \quad\rightarrow\quad 0 = I_{\text{T}}\omega_{\text{T}} + I_{\text{P}}\omega_{\text{P}} = I_{\text{T}}\omega_{\text{T}} + mR^2\left(\omega_{\text{T}} + \frac{v}{R}\right) \quad\rightarrow$$

$$\omega_{\text{T}} = -\frac{mRv}{I_{\text{T}} + mR^2} = -\frac{\left(65\text{ kg}\right)\left(3.25\text{ m}\right)\left(3.8\,\text{m/s}\right)}{1850\,\text{kg}\cdot\text{m}^2 + \left(65\text{ kg}\right)\left(3.25\text{ m}\right)^2} = \boxed{-0.32\,\text{rad/s}}$$

79. (*a*) During the jump (while airborne), the only force on the skater is gravity, which acts through the skater's center of mass. Accordingly, there is no torque about the center of mass, and so angular momentum is conserved during the jump.

(*b*) For a single axel, the skater must have 1.5 total revolutions. The number of revolutions during each phase of the motion is the rotational frequency times the elapsed time. Note that the rate of rotation is the same for both occurrences of the "open" position.

$$(1.2\,\text{rev/s})(0.10\,\text{s}) + f_{\text{single}}(0.50\,\text{s}) + (1.2\,\text{rev/s})(0.10\,\text{s}) = 1.5\,\text{rev} \quad \rightarrow$$

$$f_{\text{single}} = \frac{1.5\,\text{rev} - 2(1.2\,\text{rev/s})(0.10\,\text{s})}{(0.50\,\text{s})} = 2.52\,\text{rev/s} \approx \boxed{2.5\,\text{rev/s}}$$

The calculation is similar for the triple axel.

$$(1.2\,\text{rev/s})(0.10\,\text{s}) + f_{\text{triple}}(0.50\,\text{s}) + (1.2\,\text{rev/s})(0.10\,\text{s}) = 3.5\,\text{rev} \quad \rightarrow$$

$$f_{\text{triple}} = \frac{3.5\,\text{rev} - 2(1.2\,\text{rev/s})(0.10\,\text{s})}{(0.50\,\text{s})} = 6.52\,\text{rev/s} \approx \boxed{6.5\,\text{rev/s}}$$

(*c*) Apply angular momentum conservation to relate the moments of inertia.

$$L_{\substack{\text{single}\\\text{open}}} = L_{\substack{\text{single}\\\text{closed}}} \quad \rightarrow \quad I_{\substack{\text{single}\\\text{open}}}\omega_{\substack{\text{single}\\\text{open}}} = I_{\substack{\text{single}\\\text{closed}}}\omega_{\substack{\text{single}\\\text{closed}}} \quad \rightarrow$$

$$\frac{I_{\substack{\text{single}\\\text{closed}}}}{I_{\substack{\text{single}\\\text{open}}}} = \frac{\omega_{\substack{\text{single}\\\text{open}}}}{\omega_{\substack{\text{single}\\\text{closed}}}} = \frac{f_{\substack{\text{single}\\\text{open}}}}{f_{\substack{\text{single}\\\text{closed}}}} = \frac{1.2\,\text{rev/s}}{2.52\,\text{rev/s}} = 0.476 \approx \boxed{\tfrac{1}{2}}$$

Thus the single axel moment of inertia must be reduced by a factor of about 2.

For the triple axel, the calculation is similar.

$$\frac{I_{\substack{\text{triple}\\\text{closed}}}}{I_{\substack{\text{triple}\\\text{open}}}} = \frac{f_{\substack{\text{single}\\\text{open}}}}{f_{\substack{\text{single}\\\text{closed}}}} = \frac{1.2\,\text{rev/s}}{6.52\,\text{rev/s}} = 0.184 \approx \boxed{\tfrac{1}{5}}$$

Thus the triple axel moment of inertia must be reduced by a factor of about 5.

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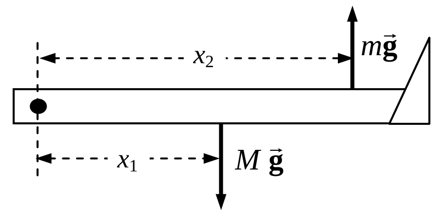
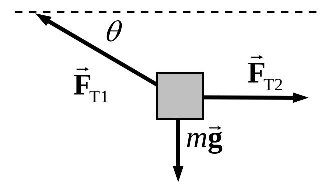
3. Because the mass m is stationary, the tension in the rope pulling up on the sling must be mg, and so the force of the sling on the leg must be mg, upward. Calculate torques about the hip joint, with counterclockwise torque taken as positive. See the free-body diagram for the leg. Note that the forces on the leg exerted by the hip joint are not drawn, because they do not exert a torque about the hip joint.



$$\sum \tau = mgx_2 - Mgx_1 = 0 \quad \rightarrow \quad m = M\frac{x_1}{x_2} = (15.0\,\text{kg})\frac{(35.0\,\text{cm})}{(78.0\,\text{cm})} = \boxed{6.73\,\text{kg}}$$

11. Using the free-body diagram, write Newton's second law for both the horizontal and vertical directions, with net forces of zero.



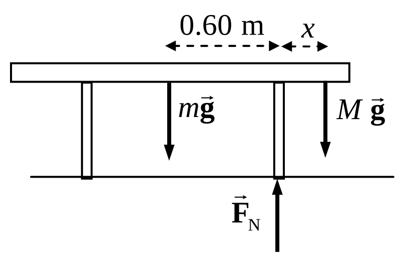
$$\sum F_x = F_{T2} - F_{T1}\cos\theta = 0 \quad \rightarrow \quad F_{T2} = F_{T1}\cos\theta$$

$$\sum F_y = F_{T1}\sin\theta - mg = 0 \quad \rightarrow \quad F_{T1} = \frac{mg}{\sin\theta}$$

$$F_{T2} = F_{T1}\cos\theta = \frac{mg}{\sin\theta}\cos\theta = \frac{mg}{\tan\theta} = \frac{(190\,\text{kg})(9.80\,\text{m/s}^2)}{\tan 33^\circ} = 2867\,\text{N} \approx \boxed{2900\,\text{N}}$$

$$F_{T1} = \frac{mg}{\sin\theta} = \frac{(190\,\text{kg})(9.80\,\text{m/s}^2)}{\sin 33^\circ} = 3418\,\text{N} \approx \boxed{3400\,\text{N}}$$

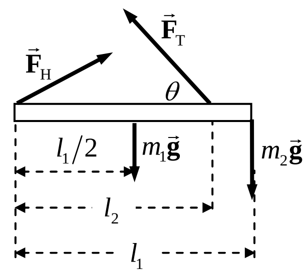
$\boxed{13.}$ The table is symmetric, so the person can sit near either edge and the same distance will result. We assume that the person (mass M) is on the right side of the table, and that the table (mass m) is on the verge of tipping, so that the left leg is on the verge of lifting off the floor. There will then be no normal force between the left leg of the table and the floor. Calculate torques about the right leg of the table, so that the normal force between the table and the floor causes no torque. Counterclockwise torques are taken to be positive. The conditions of equilibrium for the table are used to find the person's location.



$$\sum \tau = mg(0.60\text{ m}) - Mgx = 0 \quad \rightarrow \quad x = (0.60\,\text{m})\frac{m}{M} = (0.60\,\text{m})\frac{24.0\,\text{kg}}{66.0\,\text{kg}} = 0.218\,\text{m}$$

Thus the distance from the edge of the table is $0.50\,\text{m} - 0.218\,\text{m} = \boxed{0.28\,\text{m}}$.

20. The beam is in equilibrium. Use the conditions of equilibrium to calculate the tension in the wire and the forces at the hinge. Calculate torques about the hinge, and take counterclockwise torques to be positive.



$$\sum \tau = (F_T \sin\theta) l_2 - m_1 g\, l_1/2 - m_2 g l_1 = 0 \ \rightarrow$$

$$F_T = \frac{\frac{1}{2} m_1 g l_1 + m_2 g l_1}{l_2 \sin\theta} = \frac{\frac{1}{2}(155\,\text{N})(1.70\,\text{m}) + (215\,\text{N})(1.70\,\text{m})}{(1.35\,\text{m})(\sin 35.0^\circ)}$$

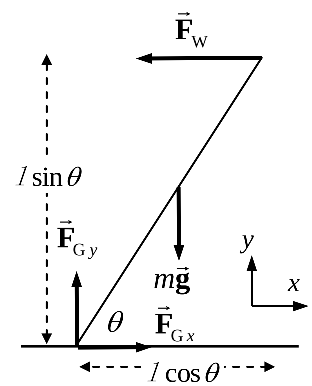
$$= 642.2\,\text{N} \approx \boxed{642\,\text{N}}$$

$$\sum F_x = F_{Hx} - F_T \cos\theta = 0 \ \rightarrow \ F_{Hx} = F_T \cos\theta = (642.2\,\text{N})\cos 35.0^\circ = 526.1\,\text{N} \approx \boxed{526\,\text{N}}$$

$$\sum F_y = F_{Hy} + F_T \sin\theta - m_1 g - m_2 g = 0 \ \rightarrow$$

$$F_{Hy} = m_1 g + m_2 g - F_T \sin\theta = 155\,\text{N} + 215\,\text{N} - (642.2\,\text{N})\sin 35.0^\circ = 1.649\,\text{N} \approx \boxed{2\,\text{N}}$$

32. Write the conditions of equilibrium for the ladder, with torques taken about the bottom of the ladder, and counterclockwise torques as positive.



$$\sum \tau = F_W \ell \sin\theta - mg\left(\tfrac{1}{2}\ell\cos\theta\right) = 0 \ \rightarrow \ F_W = \tfrac{1}{2}\frac{mg}{\tan\theta}$$

$$\sum F_x = F_{Gx} - F_W = 0 \ \rightarrow \ F_{Gx} = F_W = \tfrac{1}{2}\frac{mg}{\tan\theta}$$

$$\sum F_y = F_{Gy} - mg = 0 \ \rightarrow \ F_{Gy} = mg$$

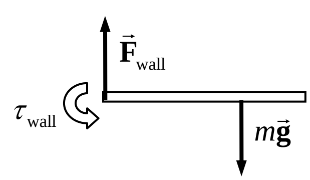
For the ladder to not slip, the force at the ground F_{Gx} must be less than or equal to the maximum force of static friction.

$$F_{Gx} \le \mu F_N = \mu F_{Gy} \ \rightarrow \ \tfrac{1}{2}\frac{mg}{\tan\theta} \le \mu mg \ \rightarrow \ \frac{1}{2\mu} \le \tan\theta \ \rightarrow \ \theta \ge \tan^{-1}\left(\frac{1}{2\mu}\right)$$

Thus the minimum angle is $\boxed{\theta_{min} = \tan^{-1}\left(\frac{1}{2\mu}\right)}$.

41. (*a*) The torque due to the sign is the product of the weight of the sign and the distance of the sign from the wall.

$$\tau = mgd = (6.1\,\text{kg})(9.80\,\text{m/s}^2)(2.2\,\text{m}) = \boxed{130\,\text{m}\cdot\text{N}\text{ , clockwise}}$$



(*b*) Since the wall is the only other object that can put force on the pole (ignoring the weight of the pole), then $\boxed{\text{the wall must put a torque on the pole}}$. The torque due to the hanging sign is clockwise, so the torque due to the wall must be counterclockwise. See the diagram. Also note that the wall must put a net upward force on the pole as well, so that the net force on the pole will be zero.

(*c*) The torque on the rod can be considered as the wall pulling horizontally to the left on the top left corner of the rod and pushing horizontally to the right at the bottom left corner of the rod. The reaction forces to these put a shear on the wall at the point of contact. Also, since the wall is pulling upwards on the rod, the rod is pulling down on the wall at the top surface of contact, causing tension. Likewise the rod is pushing down on the wall at the bottom surface of contact, causing compression. Thus $\boxed{\text{all three are present}}$.